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STOCHASTIC OPTIMIZATION OF RELIABILITY IN DESIGN AND INFLUENCE OF MODEL PREDICTION ERROR

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ABSTRACT

The knowledge about a planned system in engineering design applications is never complete. For an efficient design, all uncertainties associated with future excitation events, as well as the modeling of the system, must be explicitly accounted for. A probability logic approach provides a rational and consistent framework for quantifying these uncertainties. In this approach, probability can be interpreted as a means of describing the incomplete, i.e., missing, information about the system under consideration. This is established by characterizing the relative plausibility of different properties of the system and future excitations by probability models and leads to a *robust stochastic system design framework*. Nonparametric modeling uncertainties can be accounted for by introducing a model prediction error, i.e. an error between the response of the actual system and of the assumed model. This error can be also model probabilistically as a random variable [1] and augmented into the model parameter vector. In the context of such a probabilistic framework, the concept of robust reliability has been introduced in [2] for quantifying the stochastic performance of engineering systems. This performance is characterized by the robust probability of failure which is defined as a measure of the plausibility of the occurrence of unacceptable behavior of the system ("failure"), based on the available information, and it is given by a multi-dimensional integral over the uncertain parameter space. The utility function involved in this integral, expressing the desirability of the performance of the system response, is the indicator function, which is zero if the performance is considered acceptable and one if not.

Stochastic system design problems that involve such reliability objectives or constraints are typically characterized as Reliability-Based Design Optimization (RBDO) problems. In many RBDO applications reliability requirements are introduced as constraints on the admissible space of the design variables. In this study, though, we discuss design applications that involve the probability of failure of the system as the objective function, rather as a constraint. For complex system models, this probability can rarely

be evaluated, or efficiently approximated, analytically. To address this issue, calculation by means of stochastic simulation techniques is considered. This approach, though, involves an unavoidable estimation error and significant computational cost, features which make the associated *stochastic optimization* for finding the optimal design variables challenging. An efficient framework, consisting of two stages, is presented here for this optimization. The first stage implements a novel approach, called Stochastic Subset Optimization (SSO) [3] for performing a global sensitivity analysis. An augmented reliability problem is formulated where the design variables are artificially considered as uncertain and Markov Chain Monte Carlo techniques are implemented in order to simulate samples of them that lead to system failure. At each iteration a set with high likelihood of containing the optimal design parameters is identified using a single reliability analysis. SSO iteratively converges to a subset of the original design space that has high plausibility of containing the optimal design variables. The second stage adopts some other stochastic optimization algorithm to pinpoint the optimal design variables within that subset. Information available from the SSO stage can be used for improving the efficiency of the second stage in various ways. This can be established, for example, by (a) performing a better normalization of the search space in terms of selecting step sizes and stopping criteria for the stochastic optimization algorithm or (b) forming adaptive importance sampling densities to improve the accuracy of the evaluation of the objective function.

The influence of the model prediction error in this stochastic design process is also discussed. Different models are considered for this error and an equivalent description for the probability of failure is derived by analytically evaluating the part of the probability of failure integral that is associated with the model prediction error. This leads, ultimately, to substitution of the discontinuous indicator function (used initially as the utility function for the system performance) by a smoother function related to the specific probability model selected for the model prediction error. The implications that this alternative expression of the objective function has on the stochastic optimization are addressed.

An illustrative example is presented; it considers the optimization of a base isolation system for a three-story structure. The protection system consists of lead-rubber bilinear isolators along with supplemental viscous dampers placed at the base level. Realistic models are used for the structure as well as for describing future near-field ground motions. These models probabilistically take into account all important uncertainties of the system and, more importantly, of its excitation. The efficiency of the proposed stochastic optimization framework is illustrated. The influence of the model prediction error on the optimal design variables is also examined in the context of the example. This study reveals an important sensitivity of the optimal design to the variance parameter in the probability model selected for this error.

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