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ADAPTIVE COUPLING OF FINITE AND BOUNDARY ELEMENT METHODS IN THREE-DIMENSIONAL ELASTO-PLASTICITY

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ABSTRACT

For certain categories of problems, it is advantageous to couple the finite element method (FEM) and the boundary element method (BEM). Available FEM-BEM coupling approaches (see, e.g. references [1,2]) demand manual localization of the FEM and BEM sub-domains (defined a priori and remaining unchanged during the computation). This requires preliminary expert knowledge about the problem at hand. Besides, a predefined FEM sub-domain may result in either under-or over-estimation/incorrect choice of the nonlinear (plastic) region where the FEM is employed.

Brink et al. [3] investigated the coupling of mixed finite elements and Galerkin boundary elements in linear elasticity, taking into account adaptive mesh refinement based on a posteriori error estimators. Carstensen et al. [4] presented an h-adaptive FEM-BEM coupling algorithm for the solution of viscoplastic and elasto-plastic interface problems. Mund and Stephan [5] derived a posteriori error estimates for nonlinear coupled FEM-BEM equations by using hierarchical basis techniques. They presented an algorithm for adaptive error control, which allows independent refinements of the finite elements and the boundary elements. Doherty and Deeks [6] developed an adaptive technique for analyzing two-dimensional elasto-plastic unbounded media by coupling the FEM with the scaled boundary finite-element method. In this technique the finite-element mesh is adaptively grown to contain the nonlinear region of the problem as the solution advances. The analysis begins with an initial finite element mesh that tightly encloses the load-medium interface, whereas the remainder of the problem is modeled using the semi-analytical scaled boundary finite-element method. Load increments are applied, and if plasticity is detected in the outer band of finite-elements, an additional band is added around the perimeter of the existing mesh.

Elleithy and Langer [7] presented an adaptive FEM-BEM coupling method for two-dimensional elasto-plastic analysis. The method is capable of predicting regions that are sensible for FEM discretization. They developed a procedure that is easily automated to generate and adapt the FEM sub-domain discretization, according to the state of computation. The findings of Elleithy and Langer [7] have encouraged us to extend the work to the three-dimensional case.

This paper presents an adaptive FEM-BEM coupling method for three-dimensional elasto-plastic analysis. The FEM mesh is automatically generated over regions where plastic material behavior is expected

to develop, whereas substantial parts of the bounded/unbounded linear elastic body are approximated using the symmetric Galerkin BEM. Considerable attention is devoted to the generation and adaption of the FEM and BEM discretizations, according to the state of computation. The basic steps of implementation of our adaptive concept in three-dimensional elasto-plastic FEM-BEM coupling analysis are summarized as follows:

1. Levels of loadings are specified in order to detect regions that are sensible for FEM discretization.
2. For the specified level of loading, FEM sub-domain discretization (consequently the BEM sub-domain discretization) is automatically generated.
3. Load increments (within the specified level of loading) are applied.
4. Coupled FEM-BEM stress analysis involving elasto-plastic deformations is conducted utilizing the FEM-BEM discretization generated in step 2. For the next load increments that exceed the current specified level of loading, a repetition of steps 2-4 is required.

The proposed method is computationally efficient since it employs smaller finite element regions. The adaptive FEM-BEM coupling method has the potential advantage that it does not demand predefinition and manual localization of the FEM and BEM sub-domains. Example applications confirm the effectiveness of the proposed method.

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