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DISCRETE LIMIT ANALYSIS FOR PLATE BENDING PROBLEMS BY USING HYBRID-TYPE PENALTY METHOD

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ABSTRACT

In present paper, we have given investigation of the plate-bending problem by numerical treatment using hybrid-type penalty method (HPM). The HPM assume linear and nonlinear displacement field with rigid displacement, rigid rotation, strain, and its gradient in each sub-domain. We introduce the subsidiary condition into the framework of the variational equation with Lagrange multipliers, and hybrid type virtual work equation can be described as follows about M sub-domain and N intersection boundary:

$$\sum_{e=1}^{M}\left(\int_{\Omega^{(e)}}\boldsymbol{\sigma}:\mathrm{grad}(\delta\boldsymbol{u})dV-\int_{\Omega^{(e)}}\boldsymbol{f}\cdot\delta\boldsymbol{u}\,dV\right)-\sum_{s=1}^{N}\left(\delta\int_{\Gamma_{<s>}}\boldsymbol{\lambda}\cdot(\tilde{\boldsymbol{u}}^{(a)}-\tilde{\boldsymbol{u}}^{(b)})\,dS\right)-\int_{\Gamma_{\sigma}}\hat{\boldsymbol{t}}\cdot\delta\boldsymbol{u}\,dS=0$$

where $^{\forall}\delta\boldsymbol{u}\in\mathbb{V}$.

For this purpose accepted the Kirchhoff theory, which takes no into account the transversal shear deformation. In first step of work, we are giving the equilibrium equations for deformable body in 3D case and as boundary conditions we are giving geometrical (for displacement field) and kinetic (for surface force) boundary conditions. Secondary we apply Kirchhoff theory to make the displacement field for plate bending problem into the 3D case. For this purpose, we use quadratic form that includes rigid, linear, and nonlinear parts of displacements.

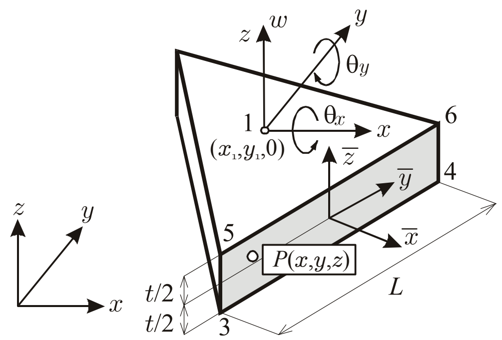

Figure 1. Mindlin plate

$$w=w_p+Y\theta_x^p-X\theta_y^p-\frac{1}{2}X^2\varepsilon_{x,z}^p-\frac{1}{2}Y^2\varepsilon_{y,z}^p-\frac{1}{2}XY\gamma_{xy,z}^p\qquad(X=x-x_1\ ,\ Y=y-y_1)$$

The parameters used in this displacement field are independently defined in each sub-domain. Since it has meaning that Lagrange multiplier is the surface force on the intersection boundary, we introduce penalty function that presents strong spring connecting each sub-domain.

$$\lambda_{<ab>}=k\delta_{<ab>}$$

It can write then subsidiary condition by the following form:

$$\boldsymbol{H}_{ab}=-\delta\int_{\Gamma_{<ab>}}\boldsymbol{\lambda}_{<ab>}^t(\boldsymbol{u}_{<ab>}^{(a)}-\boldsymbol{u}_{<ab>}^{(b)})d\Gamma=-\delta\int_{\Gamma_{<ab>}}\boldsymbol{\delta}_{<ab>}^t\boldsymbol{k}_{<ab>}\boldsymbol{\delta}_{<ab>}d\Gamma$$

Therefore, we obtain stiffness matrix by analytically integration by intersection boundary and by area of each sub-domain.

$$\boldsymbol{KU} = \boldsymbol{P} \quad \text{, where} \quad \boldsymbol{P} = \sum \boldsymbol{P}^{(e)} \quad \text{and} \quad \boldsymbol{K} = \sum \boldsymbol{K}^{(e)} + \sum \boldsymbol{K}_{<s>}$$

Discretization equation of this model becomes a simultaneous linear equation shown in above equation. Left coefficient matrix $\boldsymbol{K}$ consists of stiffness in the sub-domain and subsidiary condition on the intersection boundary for the adjustment for sub-domain.

We apply nonlinearity in penalty function such as spring system, which allow us to calculate hinge line. In case of plate bending problems yield function will be:

$$f(M) = (\frac{M_n}{M_{pn}})^2 - 1$$

If hinge line makes mechanism then we can calculate limit load. We used load incremental method, which is called r-min method as a material nonlinear analysis. We are calculating growing hinge line systematically using this algorithm for the nonlinear analysis and comparing with experimental and theoretical results. Figure 2 shows Load-Displacement curve for the simple supported square plate with distributed load. Present method can obtain same limit load with theoretically plastic analysis value. Figure 3 shows final hinge lines calculated for the same plate. It is similar with hinge line obtained theoretically.

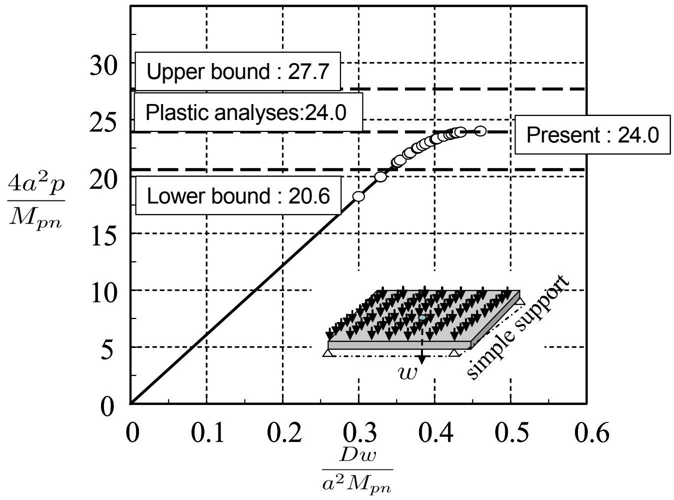

Figure 2. Load-Displacement curve

After comparison of theoretical solution and HPM results for limit load and hinge lines in case of several examples, we can see that we obtain the same limit load and similar hinge lines. As a result, we can conclude that HPM corresponds to all requirements for solving problems such as plate bending.

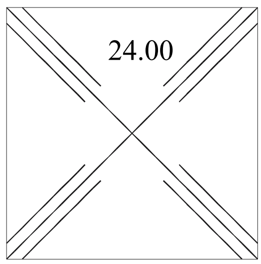

Figure 3.Hinge lines

REFERENCES

[1] T. Kawai, *My challenge in the development of a mixed variational method in solid mechanics,* Applied Mechanics Reviews, Transaction of the ASME, Vol. 60, (2007), pp.51-64.

[2] N. Takeuchi, et al, *Material nonlinear analysis by using discrete model applied penalty method in hybrid displacement model,* Transactions of JSCES, Paper No 20010002, (2001), pp.52-62.

[3] H. Ohki, N. Takeuchi, *Upper and low bound solution with hybrid-type penalty method,* Transactions of JSCES, Paper No 20060020, (2006), pp.1-10.

[4] A. Vardanyan, N. Takeuchi, *Discrete Analysis for Plate Bending Problems by Using Hybrid-type Penalty Method*, Proceedings of the APCOM'07 in conjunction with EPMESC XI (2007).