8th. World Congress on Computational Mechanics (WCCM8)
5th European Congress on Computational Methods in Applied Sciences and Engineeering (ECCOMAS 2008)
June 30 –July 5, 2008
Venice, Italy

Mathematical Modelling and Numerical Simulation of Blood Flow in Compliant Vessels

*** Mária Lukáčová-Medviďová[1], Anna Zaušková[1,2]**

[1] Institute of Numerical Simulation, Hamburg University of Technology
Schwarzenbergstrasse 95, 21073 Hamburg, Germany
lukacova@tu-harburg.de
www.tu-harburg.de/mat/hp/lukacova

[2] Department of Mathematical Analysis und Numerical Mathematics, Comenius University Bratislava
Mlynská dolina 84248, Bratislava, Slovak Republic
zauskova@tu-harburg.de
http://hore.dnom.fmph.uniba.sk/˜ zauskova

Key Words: *non-Newtonian fluids, fluid-structure interaction, shear-thinning fluids, hemodynamical wall parameters, existence and uniqueness of weak solution*

ABSTRACT

Blood is a complex, non-Newtonian fluid-solid mixture which consists of deformable cells, e.g. red blood cells (RBCs) that are suspended in an essentially Newtonian plasma. Blood's microstructure, in particular the RBCs, imply the non-Newtonian behaviour. The blood exhibits the *shear-thinning* property, i.e. breakup of RBCs at low shear, *viscoelasticity*-due to the cell deformation and cell aggregation and shear anisotropy-due to the alignment of cells with the flow direction [4,5,6]. In the present work we use the macroscopic model that reflects the shear-thinning blood property. In particular we work with the power-law type model of the Carreu and the model with Yeleswerapu viscosity, see [3,5,6].

Our aim is to study the *blood flow in elastic vessels*. In practice the deformation of elastic artery wall is not negligible and can lead to localized reversal flow which implies a formation of arteriosclerotic plaques and stenosis - inner lumen constriction of the vessels. The mathematical model is based on the fluid-structure interaction between the shear-thinning fluid and elastic vessel. The deformation of the blood vessels is modelled by the generalized string equation which yields a second order differential equation for the wall displacement η. The interaction between fluid and structure is twofold. First, the structure enforces a non-homogeneous Dirichlet-type boundary condition on the vessel wall $\mathbf{u}(x,y,t) = \eta_t(x_1,t),\;\; (x,y) \in \Gamma^{wall}(t)$. Moreover, the fluid enforces the movement of the vessels and thus we have in the string equation the forcing term induced by the fluid stress tensor. The coupled problem is solved iteratively by means of the strong coupling between the fluid and structure, see, e.g., [3,7] for more details.

We present our recent theoretical results of existence and uniqueness of the weak solution for shear-dependent flows in compliant vessels [2]. The existence result for Navier-Stokes equation in a time dependent domain obtained in [1] have been generalized for the shear-depended fluids. Having in the non-Newtonian case an additional non-linear viscous term, new techniques have been used to obtain

necessary estimates. Finally we present results of numerical simulations for blood flow in stenotic vessels, see Figure 1. Study of hemodynamical wall parameters confirms the importance of using the non-Newtonian rheology in order to get reliable predictions of arterosclerotic plugs.

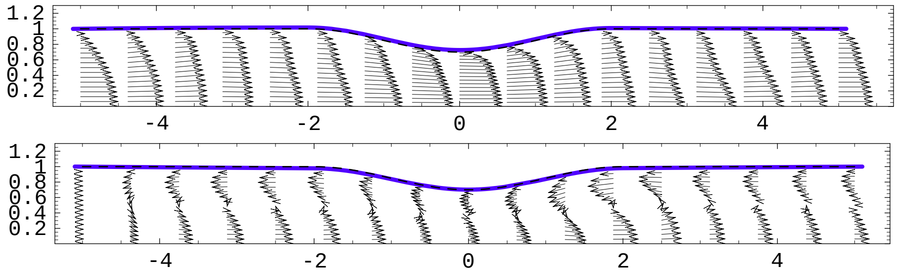

Figure 1: Velocity field in two time instances with the Carreau non-Newtonian model for viscosity, $RE = 60, t = 0.36\ s,\ t = 0.96\ s$.

REFERENCES

[1] J. Filo, A. Zaušková. “2D Navier-Stokes equations in a time dependent domain with Neumann type boundary conditions”, accepted *Journal of Mathematical Fluid Mechanics*, 2006.

[2] M. Lukáčová-Medviďová, A. Zaušková, “Mathematical and numerical modelling of complex flow in compliant vessels”, Preprint Technische Universität Hamburg-Harburg, 2008.

[3] M. Lukáčová-Medviďová, A. Zaušková, “Numerical modelling of shear-thinning non-Newtonian flow in compliant vessels”, accepted *Int. J. Num. Meth. Fluids*, 2007.

[4] L. Nadau, A. Sequeira, “Numerical simulations of shear dependent viscoelastic flows with a combined finite element—finite volume method” *Computers and Mathematics with Applications* (to appear).

[5] K.K. Yeleswarapu, *Evaluation of continuum models for chatacterizing the constitutive behaviour of blood* Pittsburg, 1996.

[6] K.K. Yeleswarapu, M.V. Kameneva, K. R. Rajagopal, J. F. Antaki, The flow of blood in tubes: theory and experiments. *Mech. Res. Comm.* Vol. **25(3)**, 257-262, 1998.

[7] A. Zaušková. *2D Navier-Stokes Equations in a Time-Dependent Domain*, Dissertation Thesis, Comenius University, Bratislava, 2007.