Invariant manifold identification from phase space trajectories

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The identification of basins of attraction is of fundamental importance in the study of dissipative dynamical systems. These basins are delineated by the invariant manifolds of the system. Thus, by identifying the location of the invariant manifolds, we are able to identify the basins of attraction, which may have fractal structures. There exists a large body of literature for the identification of invariant manifolds in two and three dimensional flows [1,2,3] when the underlying flow field is given. However, there are few, if any, methods for the detection of these manifolds when only phase space trajectories are given, which is typical in vibration problems. We provide three methods capable of identifying the stable and unstable manifolds of a system given only phase space trajectories and provide a procedure for implementing existing methods, where possible. Data requirements for the detection methods are also considered.

The stochastic interrogation method [4] is used to obtain a well populated phase space to construct our data set. However, this may not be needed for a chaotic system. Our first method consists of tracking a cloud of nearest neighbor trajectories in time and quantifying its deformation (similar to Bowman 1993 preprint). Maxima in deformation under forward(backward) time integration correspond to the location of the stable(unstable) manifold. The second methods compares a true trajectory to one calculated using a local linear model (LLM) of N nearest neighbor trajectories. The LLM approximates the trajectories accurately away from the manifolds, but small errors are amplified near the manifolds. The manifolds then correspond to the maxima in the error between true and LLM trajectories. The final method is based on the concept of phase space warping [5]. We compare phase space trajectories of the system for two sets of slightly different parameters. The small change in parameters slightly shifts the location of the manifolds, which significantly alters the trajectories located near the manifolds. Thus, by locating maxima in trajectory errors between the two parameter sets, we again locate the stable and unstable manifolds. Proof of concept calculation for the periodically forced Duffing's Oscillator (Figure) show the above methods are indeed capable of identifying invariant manifolds given sufficient data. The methods are also shown to be applicable to other systems, such as fluid flows.

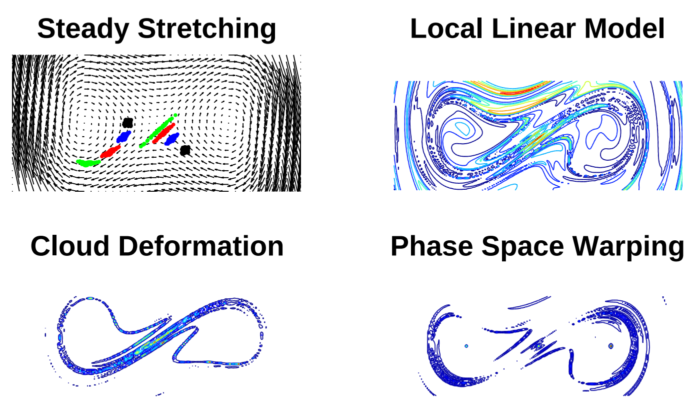

Figure 1: Examples of manifold detection methods using forced duffings oscillator. Also shown is the deformation of a cloud of nearest neighbors for the steady Duffing's Oscillator. Notice that near the manifolds the cloud deforms much more than away from the manifolds.

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