8th. World Congress on Computational Mechanics (WCCM8)
5th. European Congress on Computational Methods in Applied Sciences and Engineering (ECCOMAS 2008)
June 30 – July 5, 2008
Venice, Italy

TRANSPARENT BOUNDARY CONDITIONS FOR ANISOTROPIC ELASTODYNAMICS

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Key Words: *Non-reflecting boundary conditions, quasi-analytic transparent boundary conditions, anisotropy, elastodynamics, VTI.*

ABSTRACT

Generation of reliable non-reflecting boundary conditions (NRBC) on artificial open boundaries in anisotropic media is a crucial task for correct numerical modelling in elastodynamics. Recent results [1], [2] of using the popular perfectly matched layers approach for generating NRBC show that it can fail: natural physical constrains appear clearly in some kinds of anisotropy.

We propose an approach that permits to generate numerically the desired NRBC for any kind of anisotropy. It uses ideology of transparent boundary conditions (TBC) [3]. As two representative examples we consider the equations of elastodynamics for displacements in cylindrical two-dimensional (r,θ) and (r,z) geometries, respectively, [4], [5]. Numerical modelling shows excellent transparency of open boundaries with our NRBC for all tested cases, including long time calculations with wave fronts running several tens radiuses of computational domain.

Our approach uses the Green's function for governing equations in the exterior domain, which is obtained numerically because of variable coefficients in both directions. To calculate it with admissible computational costs we introduce Fourier basis functions $\{\psi_k\}_{k=1}^N$ on the open boundary while representing displacements and their normal derivatives. Actually the Green's function is the $N \times N$ matrix $\{G_{k,l}(t)\}$, $t > 0$, of time-dependent Fourier coefficients: given an "elementary source", i.e. the instantaneous displacements ψ_l at $t = 0$, the l-th column of the matrix supplies the "response" $\sum_k G_{k,l}(t)\psi_k$ – normal derivatives of displacements on the open boundary.

The algorithm of calculating $\{G_{k,l}(t)\}$ consists of two main parts. First, after the Laplace transform of governing equations with respect to time, we calculate Dirichlet-to-Neumann maps of auxiliary well-posed external elliptic problems parameterized by the dual variable s. Discretization of these elliptic problems is made by using second order finite differences in the r-direction and the Galerkin's approach with the basis $\{\psi_k\}_{k=1}^N$

for the transversal direction (θ or z for (r,θ) or (r,z) geometry, respectively). Each problem is solved numerically, the Richardson extrapolation with respect to r-direction is applied increasing the approximation order up to 8. The number of such problems is the product $N_S N$ where N_S is the length of the dual variable set $\left\{s_j\right\}_{j=1}^{N_S}$. In the second part of the algorithm each entry of this resulting matrix $\left\{\hat{G}_{k,l}(s_j)\right\}$ is approximated by rational functions with respect to s in order to represent time-dependent components of our TBC operator in terms of sums-of-exponentials after the inverse Laplace transform $G_{k,l}(t) = L^{-1}\left[\hat{G}_{k,l}(s_j)\right]$.

Also we report preliminary results concerning a generalization of the approach for the case of layered media in (r,z) geometry.

The work is supported by the RFBR grant No. 07-01-00476.

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