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Higher Order Approximation in the Meshless Finite Difference Method – state of the art

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ABSTRACT

Meshless Finite Difference Method (MFDM, [1]) belongs to the wide class of meshless methods, one of the basic approaches used to analyse boundary value problems nowadays. It is based on the arbitrary irregular cloud of nodes and the Moving Weighted Least Squares approximation (MWLS, [1]). The paper focuses on a recent modification of the higher order (HO) approximation, introduced in [5] and further developed in [6-9] as well as on a'posteriori error analysis in the MFDM.

The present state of the art indicates several possible approaches that may be used to improve FD solutions. Increasing the number of nodes is the most obvious way. The number of nodes may rapidly increase then, whereas the order of the approximation remains unchanged. The other approach relies on raising the approximation order of the searched functions. The concept of the HO approximation in the MFDM is based on considering additional correction terms yielded by the Taylor expansion of unknown function [5] rather than introducing new nodes into simple FD operators, as proposed in [2]. The expansion of the unknown function u into the Taylor series produces additional higher order correction terms Δ, which may involve the HO derivatives. They are calculated using appropriate formulae composition inside the domain, including singularities as well as jump terms of the function u, and / or its derivatives. Near the boundary they may need special treatment, e.g. use of the multipoint approach [3]. Correction terms modify the right hand sides f of the MFD equations. The final HO solution $u^{(H)}$ depends only on the truncation error of the Taylor series. The whole solution process needs only two steps, both with the same MFD operator L. This way the total number of nodes in the mesh may be reduced, without compromising the quality of the FD solution, which in fact is improved by raising the rank of the local approximation. The HOA approach may be combined with a multigrid iterative procedure [2,8].

Special emphasis is laid upon improved a'posteriori error estimation, where the HO terms may be used in several ways [7-9]. Checking the local solution error e or the local residual error r at specially chosen points of the domain is considered first [1].

Error estimation may use the HO terms: for both the solution error $e \approx u^{(H)} - u$ and the residual error $r \approx Lu^{(H)} - \Delta - f$. Various integral type error estimators may be used, like those developed in the Finite Element Method including hierarchic or smoothing $\|e\|$ as well as residual type $\|r\|$ ones. Here $\|\cdot\|$ denotes appropriate integral norm. HO approximation done here by means of the HO correction terms may raise the approximation order of the standard solution up to 2p in the HO case. Numerous tests done show that such approach provides the best estimation of the exact solution, when compared to smoothing techniques widely used in the FEM [4].

In order to speed up the analysis of the considered boundary value problem on a set of meshes (usually irregular) the multigrid approach [1,2,8] is used. A combination of the adaptive multigrid approach [1] and HO correction terms one [8] is considered here.

The approach was tested on many 1D and 2D benchmark problems, giving very promising results, towards solving large b.v. problems. Especially investigated were applications of the HO terms to a'posteriori error estimation, and a comparison of the calculation times used for multigrid schemes and versus the standard approach.

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