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Parameter Estimation for Noisy Models using Total Least Squares

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ABSTRACT

Parameter Estimation problems as usually presented fall into two groups: those that use simulated data; and those that use real (experimental) data. For both groups the underlying assumption is that the model of the system is fundamentally correct. Papers using simulated data are usually aimed at demonstrating new or different estimation approaches. This fundamental assumption is clearly valid for simulated data since the 'measurements', i.e., the simulated data, almost always consist of the model response corrupted by noise. For real problems, the assumption need not be correct and the parameter estimates are affected both by measurement noise and by inexact and/or incomplete models. Even if the model is inherently correct, i.e., correct in the average, the parameters or boundary and initial conditions may not be known precisely and uncertainty in these values will impact the resulting estimates.

We term a model as being 'noisy' when there is uncertainty in the model as it is employed. Such uncertainty may arise from discretization errors in the numerical solution, from noise in boundary conditions, and from the stochastic behavior of material properties. In addition, we include statistical uncertainty in nominally constant parameters.

Let the response be given by y=M(x,t,p) where x and t represent location and time and p are parameters of the model. In general, the model M(x,t,p) does not have a closed form solutions and is evaluated numerically. The paper explores the effects of uncertainties in x, t, and p from the point of view of "errors-in-variables" models. As a simple example, consider fitting a model of a straight line to data, yt=A xt where yt is the true response, xt is the independent variable and A xt is the model. We assume that the response is corrupted, i.e., y=yt + e. The usual least squares approach fits y=A xt +e by assuming that the only error is in the measured response, y=yt+e. With a noisy model, we augment the model as y=A z+b+e where b is the model noise and in addition we consider that xt itself is inaccurately measured, z=xt+c. The model noise, b, is due

either to numerical errors in solving the model, to uncertainty in model parameters or to both and to the effect of inaccuracies in x, e.g., A c. Fitting such noisy models can be done using the total least squares (TLS) approach. Unfortunately this approach requires that the statistical characteristics of the different noise terms, e, c, and b, are known. For the usual least squares approach, the mean and the standard deviation of the measurement noise, e, are estimated from the residuals. With TLS, this proves to be a much more complex task.

One important feature of TLS is that not only are the values of y corrected, but also those of x and the model M(x,t,p). Thus one learns not only a better estimate of what the true values of the system response should be but also a better estimate of how the model should behave. For example, in the fitting of the straight line we obtain improved estimates of yt and xt. Of course this comes at a cost. The problem has grown from one involving the single value, A, to one involving N+1 values of A and xt. When estimating a m parameters in a non linear model using least squares we have to solve only m equations in an iterative manner and the algebra is straightforward. The effects of varying sensitivities on the estimated parameters are easy to quantify. Using TLS we must solve a much larger problem – this is especially true when there are several responses measured at high frequencies. Even more disconcerting is that depending upon the constraints of the model there need not be a solution to this problem. However, when there is a solution, one learns much about both the specific problem and the fundamental nature of the model.

The paper treats a transient thermal problem for estimating the thermal conductivity and the boundary convective heat transfer coefficient and a packed bed energy storage system for estimating the heat transfer between the bed and the air. The paper begins with a discussion of the effect of numerical noise in predicting the system response, e.g., A xt + b, which is known to cause a bias in the estimate of the parameters, in this case A. The model is then assumed to have noise in the properties, such as the conductivity and the packing density, which combines with the numerical noise to increase the bias. Finally models in which there is boundary condition noise due to the stochastic variation in the convective heat transfer coefficient or the contact resistance between the bed and the container walls are studied.