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STOCHASTIC DATA ASSIMILATION WITH A KARHUNEN-LOEVE / POLYNOMIAL CHAOS STATISTICAL REDUCTION

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ABSTRACT

This contribution focuses on estimating a representation of the statistical properties of a stochastic process. More specifically, we aim at estimating a statistical representation of the sea bottom displacement field in earthquakes. The sea surface data is available through, say, satellite observations and an inverse method is used to retrieve the corresponding time-varying sea bottom displacement field for each earthquake event. From a collection of these observations, the goal is to estimate a statistical representation of the displacement field S.

The approach is two-fold: first, for each realization θ_S (earthquake event), one needs to retrieve the displacement field $S(x, t, \theta_S)$ giving rise to the observed time-varying sea surface field. Second, with a collection of n_{obs} identified displacement fields and in the spirit of [1], determine a representation of its statistical properties.

The first step consists in retrieving the input of a process given its output (observations). As the physical system is only poorly known and characterized, it is subjected to uncertainties (sea depth field, salt content, sea temperature profile, etc.) and this step thus constitutes a stochastic inverse problem. An additive Schwarz preconditioned spectral elements formulation is used to solve the primal and dual solutions of the stochastic Shallow Water Equations. This optimization framework combined with a L-BFGS strategy leads to the determination of S.

Using a Karhunen-Loeve decomposition, approximating the random field reduces to determining a finite set of N uncorrelated random variables $\phi_i(\theta_S^{(n)})$, $i = 1, \ldots, N$, $n = 1, \ldots, n_{obs}$, hereafter simply denoted $\phi_i^{(n)}$. One then needs a representation of the different random variables ϕ_i. In this work, use is made of their spectral approximation based on the so-called Polynomial Chaos (PC) decomposition [2] where the Karhunen-Loeve coefficients ϕ_i are modeled as random quantities defined on an abstract probability space $(\Theta, \mathcal{B}, dP)$. The approximation is done at the level of the statistic behavior and not for a particular realization. This results in identifying the coefficients $\alpha_j^{(i)}$, $j = 0, \ldots, P$, $i = 1, \ldots, N$ of the PC decomposition through an inverse-like method where the goal is to find $\alpha_j^{(i)}$ such as the surrogate $\widetilde{\phi}_i(\theta) = \sum_{j=0}^{P} \alpha_j^{(i)} \Psi_j(\xi(\theta))$ is the closest to $\phi_i^{(n)}$, $n = 1, \ldots, n_{obs}$ in the statistical sense: this is the

maximum likelihood approach. Here ξ is a vector of N_ξ iid Gaussian random variables with zero mean and unit variance and $\theta \in \Theta$.

To solve the optimization problem resulting from the maximum likelihood approach, the likelihood function $\mathscr{L}$ is defined as

$$\mathscr{L} \equiv \prod_{n=1}^{n_{obs}} \mathscr{E}_\Theta \left\{ \delta \left[\phi^{(n)} - \widetilde{\phi}(\theta) \right] \right\} = \prod_{n=1}^{n_{obs}} \int_\Theta \delta \left[\phi^{(n)} - \widetilde{\phi}(\theta) \right] \, dP(\theta) \tag{1}$$

and its integrand is regularized yielding

$$\widetilde{\mathscr{L}} \equiv \prod_{n=1}^{n_{obs}} \int_\Theta \widetilde{\delta} \left[\phi^{(n)} - \widetilde{\phi}(\theta) \right] \, dP(\theta) \tag{2}$$

where the Dirac distribution δ is replaced by a function $\widetilde{\delta} \in H^1(\mathbb{R}^N)$ and $\mathscr{E}_\Theta$ denotes the mathematical expectation.

This regularized expression of the likelihood allows for an explicit form of its Gâteaux derivative and a gradient-based optimization strategy can then be used to find the optimal coefficients vector $\alpha^* \equiv \arg\max_{\alpha \in \Omega_\alpha} \widetilde{\mathscr{L}}(\alpha)$.

Numerical examples of the whole solution method will be presented to demonstrate the ability of the current approach to efficiently estimate a statistical representation of a stochastic process from experimental data. As a first glimpse of the results, Fig. 1 shows the sea surface computed both from the actual displacement field S (left) and from the one identified through stochastic inverse method (right). The good agreement demonstrates the effectiveness of the identification step.

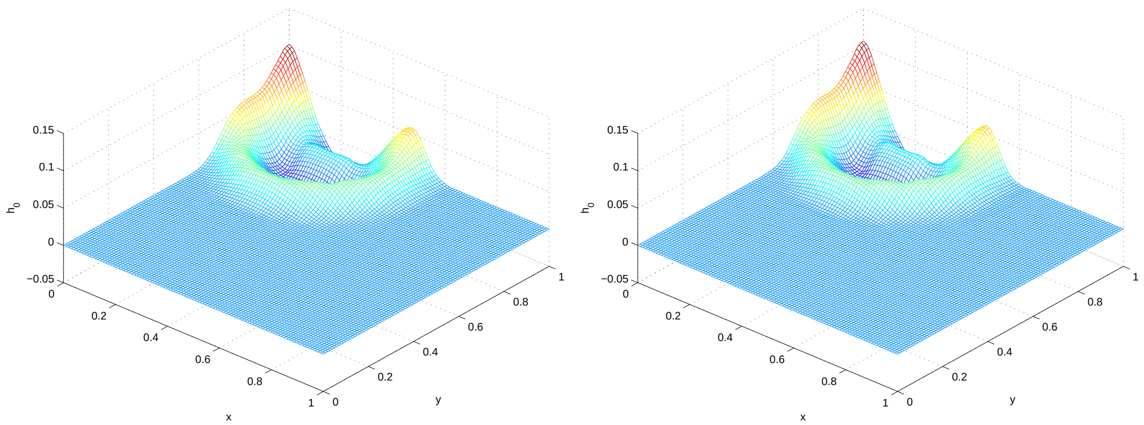

Figure 1: Mean sea surface field at intermediate time ($t = 2400$ adimensional units) for the actual displacement field (left) and the inverse-computed field (right). A good agreement can be observed. The ocean is here considered to be a 1000 kilometer wide square of uncertain depth of mean = 1000 m and standard deviation = 200 m following a uniform distribution.

REFERENCES

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