AN EXTENDED LEVEL SET METHOD IN STRUCTURAL OPTIMIZATION

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ABSTRACT

The paper is concerned with the numerical solution of a structural optimization problem for an elastic body in unilateral contact with a rigid foundation. The contact with a given friction, described by Coulomb law, is assumed to occur at a portion of the boundary of the body. The displacement field of the body in unilateral contact is governed by an elliptic variational inequality of the second order. The results concerning the existence and finite - dimensional approximation of solutions to contact problems are given, among others, in [4]. The structural optimization problem for the elastic body in contact consists in finding such topology of the domain occupied by the body and the shape of its boundary that the normal contact stress along the boundary of the body is minimized. It is assumed, in a case of shape optimization problem, that the volume of the body is constant.

Shape optimization of contact problems is considered, among others, in [4,6,9], where necessary optimality conditions and numerical results are provided. The material derivative method is employed in monograph [6] to calculate the sensitivity of solutions to contact problems as well as the derivatives of domain depending functionals with respect to variations of the boundary of the domain occupied by the body. Topology optimization deals with the optimal material distribution within the body resulting in its optimal shape [1,2,7,10]. The topological derivative is employed to account variations of the solutions to state equations or shape functionals with respect to emerging of small holes in the interior of the domain occupied by the body. The notion of topological derivative and results concerning its application in optimization of elastic structures are reported in many papers (see references in [10]).

In structural optimization the level set method [8] is employed in numerical algorithms for tracking the evolution of the domain boundary on a fixed mesh and finding an optimal domain. This method is based on an implicit representation of the boundaries of the optimized structure, i.e., the position of the boundary of the body is described as an isocountour of a scalar function of a higher dimensionality. While the shape of the structure may undergo major changes the level set function remains to be simple in its topology. The evolution of the domain boundary is governed by Hamilton - Jacobi equation. The speed vector field driving the propagation of the level set function is given by the Eulerian derivative of the cost functional with respect to the variations of the free boundary. Applications of the level set methods in structural optimization can be found, among others, in [3,6]. Recently, in the series of papers

[5,11] different numerical improvements of the level set method employed for the numerical solution of the structural optimization problems are proposed and numerically tested.

This paper deals with topology and shape optimization of an elastic contact problems. The structural optimization problem for elastic contact problem is formulated. Shape as well as topological derivatives formulae of the cost functional are calculated using the material derivative [9] and the asymptotic expansion [10] methods, respectively. These derivatives are employed to formulate necessary optimality condition for simultaneous shape and topology optimizationand to calculate descent direction in the numerical algorithm. Level set method extended rather than classic is used to describe the position of the boundary of the body and its evolution on a fixed mesh. This evolution is governed by Hamilton - Jacobi equation where the shape gradient of the cost functional is employed as a velocity field. The proposed method is based on the application of radial basis functions and new scheme of the extension of velocity field from the boundary of the domain into the whole domain. Level set based numerical algorithm for the solution of the shape or topology optimization problem is proposed. The finite element method is used as the discretization method. Numerical examples are provided and discussed.

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