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NUMERICAL SIMULATION OF HEAT CONDUCTION IN ELEMENTARY RANDOM MEDIA

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ABSTRACT

Stimulated by the increasing requirement of modelling practical random media as well as the rapid development of computational power and data storage, in the past decade, uncertainty modelling has attracted considerable attention in the community of computational mechanics. A highly recommended introductory textbook, with historical insights, is Ghanem [1]. Some recent developments of stochastic finite element modelling can be found in [2].

This work focuses on heat conduction in random media which is relevant to a wide variety of phenomena in nature and industrial processes. A partial list of such phenomena includes transport in porous media such as soils and rocks, and diffusion through packed/fluidized beds in chemical engineering and nuclear engineering.

First, based on existing developments in stochastic finite elements, we proposed a new material model, termed *elementary random media*, for the modelling of practical random media such as soils, rocks and composite materials with random inclusions. The macro-scale properties of elementary random media, including regularity, stationarity and ergodicity etc., are also discussed in detail. Then, the recently developed Fourier-Karhunen-Loève discretization scheme [3, 4] is adopted to decompose the randomness of the conductivity tensor so that the standard finite element discretization can be applied to construct a stochastic system of linear algebraic equations. For large scale random structures containing a large number of DOFs and a large number of random variables, the solution of the corresponding stochastic linear system is very challenging and remains outstanding. Hence, based on the novel joint diagonalization solution strategy [5], we developed a new numerical solver which is much more efficient in dealing with large scale stochastic linear systems. Several numerical examples are presented to demonstrate the performance of the proposed simulation framework.

REFERENCES

[1] R.G. Ghanem and P.D. Spanos, *Stochastic finite elements – a spectral approach*, Revised edition, Dover Publications, New York, 2003.

[2] Edited by G.I. Schuëller, Special issue on computational methods in stochastic mechanics and reliability analysis, *Comput. Methods Appl. Mech. Engrg.* Vol. 194, pp. 1251 – 1795, (2005).

[3] C.F. Li, Y.T. Feng, D.R.J. Owen and I.M. Davies, Fourier representation of random media fields in stochastic finite element modelling, *Engineering Computations*, Vol. 23, pp. 794-817, (2006).

[4] C.F. Li, Y.T. Feng, D.R.J. Owen, D.F. Li and I.M. Davies, A Fourier-Karhunen-Loève discretization scheme for stationary random material properties in SFEM, Int. *J. Numer. Meth. Engng.*, In press (available online).

[5] C.F. Li, Y.T. Feng and D.R.J. Owen, Explicit solution to stochastic system of linear algebraic equations $(\alpha_1\boldsymbol{A}_1 + \ldots + \alpha_m\boldsymbol{A}_m)\boldsymbol{x} = \boldsymbol{b}$, *Comput. Methods Appl. Mech. Engrg.* Vol. 195, pp. 6560-6576, (2006).