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A BAYESIAN FRAMEWORK FOR ORTHOTROPIC ELASTIC CONSTANTS IDENTIFICATION ACCOUNTING FOR BOTH ERROR AND VARIABILITY

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ABSTRACT

Identifying parameters of a model using experimental data has been extensively studied in various areas, including for determining elastic material properties from strain measurements or modal vibration data. In order to find the parameters that make the model agree best with the experiments, the most widely used method is based on minimizing the least squares error between the experimental data and the model predictions.

The least squares method leads however to suboptimal results under a variety of conditions. In a previous study by the same authors [1] it has been shown on two simple test problems that the least squares method can be negatively affected by different magnitude of the experimental data, different uncertainty and correlation among the experimental data. Multiple sophistications to the least squares method exist in the literature, addressing these issues.

Another natural way to address the previous shortcomings is through statistical frameworks, based on maximum likelihood or on Bayes' rule. The Bayesian framework is more general since it can include prior knowledge. Isenberg proposed the application of the Bayesian framework for parameter estimation in 1979 [2] and several articles have been published since on the application of this approach to frequency or modal identification in particular, i.e. identifying material properties from vibration test data [3],[4].

The statistical nature of the Bayesian approach implies that it can intrinsically handle the shortcomings discussed earlier on the classical least squares method. However it also implies numerical and computational cost issues since we usually have to carry out large numbers of simulations.

Different types of uncertainty can be considered in a Bayesian framework, with error and variability representing two major categories. By error we mean either measurement error, which includes systematic as well as random error (or noise) or modeling error,

that is, the error induced by using a certain model to represent the actual experiment. Variability on the other hand relates to uncertainty of fixed input parameters of our model, such as dimensions of the plate or plies, exact fiber volume fraction or fiber orientation. The uncertainty in the knowledge of these fixed parameters, unlike the error uncertainty, can be accurately propagated through the model to determine its effects on the identified parameters. However this propagation process is particularly computationally expensive.

Previous studies such as [4] that applied the Bayesian approach for material properties identification from vibration test data of a beam considered error but not the more expensive variability. In our previous study [1] we did consider both error and variability however the Bayesian approach was applied on a simple vibration test problem for which we used an analytical model. The aim of this previous study was to illustrate on an example where computational issues do not cloud the process how the Bayesian approach can handle variable uncertainties as well as correlation in the measurements.

In the current study we are interested in what happens when we apply a Bayesian updating procedure, accounting for both error and variability, to an actual identification problem, where inevitably we have to deal with numerical and computational cost issues. The chosen application consists in identifying orthotropic elastic properties from frequency response data of a free vibrating composite plate. The focus of the study is on the numerical side so we chose to use experimental results already available in the literature [5].

The final contribution will discuss the difficulties of accounting for both error and variability in a Bayesian approach and the solutions retained to tackle this problem, applied on the identification of orthotropic material properties from vibration test data.

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