

Shell based robust numerical procedures for the solution of real-life problems

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ABSTRACT

The modeling of several real-life complex problems requires the use of thin structures. It is important to study and develop effective and reliable shell elements, especially for coupled problems where the validation of the entire method needs the efficiency of each component.

All the numerical procedures presented use “general shell elements” either the MITC shell elements [1] or the “3d shell elements” [2]. All these elements prevent locking which is an important feature in industrial applications.

The main interest of the 3d-shell elements is to use 3D energy without prior modification, unlike classical general shell elements which require the use of a plane stress assumption and of an accordingly modified energy formulation. This is particularly effective when using non-linear stress-strain laws as the Ogden-Ciarlet-Geymonat energy density. These elements have standard degrees of freedom and thus are well suited for multi-layered structures and particular coupling situations.

In this context we are interested in the interaction between an incompressible fluid and an elastic structure. As the target application is blood flows through large arteries, a strongly coupled method which ensures a well-balanced energy transfer between the fluid and the structure is mandatory. It is well-known that a heterogeneous domain-decomposition approach is well suited to solve such problems [4].

A domain decomposition method allows the use of different solvers for the fluid and the structure problem, each with its own schemes [3]. The use of two different solvers has several established benefits such as re-usability of existing codes and flexible choice of the numerical methods adapted to each sub-problem. In addition, this approach allows to design scalable approaches for large, memory- and time-intensive problems by using multi-level state of the art algorithms for both fluid and solid solvers. We will focus on the solution of the solid problem. For the blood flow simulations as the wall arteries are thin it is convenient to use shell elements; they accurately describe the geometry. For simple models of the wall arteries MITC4 general shell elements are used. If we aim to consider more realistic models

of the wall with several layers (the intima, media and adventitia) then general 3d shell elements are more convenient, but the cost of the solid problem increases. A Newmark algorithm is used for the time integration combined to a Newton algorithm for the solution of the nonlinear problem arising at each time step. As the local tangent problems for the structure are very ill-conditioned, direct methods are preferred to solve them, but this dramatically increases the memory requirements. To overcome this difficulty, when dealing with large problems, in our simulations we introduce a second level of domain decomposition by using a balanced Neumann-Neumann method inside the structure. Numerical examples will show the flexibility of the method.

Another problem of interest is the study of multi-materials with strong interfaces. The first simplified model considers two elastic bodies connected by a strong thin material layer whose stiffness grows [5]. This problem is difficult to solve directly by 3d models because the thin layer implies a very large mesh and moreover the difference in elastic coefficients produces instabilities. Introducing a variational limit problem allows to substitute the thin layer by a “material surface” which, in this instance, will be modeled by a shell. An alternative to the direct solution of this problem is to introduce an appropriate domain decomposition type technique. This work is in progress.

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