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NUMERICAL MODELING OF THE HUMAN CARDIOVASCULAR SYSTEM

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ABSTRACT

Mathematical investigation of the human circulatory system has drawn increasing attention in recent years, also because cardiovascular diseases represent one of the leading causes of natural death in developed Countries.

One of the most frequent vascular pathologies, the atherosclerotic thickening, typically occurs in preferential sites, such as in outer wall of vessel bifurcations (like in the internal carotid artery), inner wall of curved segments (like in coronary arteries), and anastomotic junctions (e.g. downstream a coronary by-pass). More recently, this was related to peculiar behavior of specific fluid dynamical quantities, notably the wall shear stress, which is defined as the normal derivative of the tangential flow velocity at the internal vessel walls. More precisely, it has been conjectured that critical flow zones are those where wall shear stress is low but its rate of variation in time is high. This dynamical flow behavior is measured by the so-called oscillatory shear index, whose determination requires solving a truly three dimensional flow problem in real geometries. More generally, mathematical models based upon partial differential equations are nowadays recognized to be the matter of choice if one aims at reproducing the complex features exhibited by blood flow under either physiological or pathological conditions.

Modeling the flow behavior is not the only task to fulfill prior to carry out numerical simulations. Indeed, from one hand one has to construct the computational domain, starting by clinical data such as MRI (magnetic resonance imaging), digital angiography or computed tomography (CT scans). By the same tools the boundary conditions (on fluid velocity and/or fluid pressure) need to be provided at the inlet and outlets of the segment of arteries that we want to address for the numerical simulation.

Another issue that we have to face before carrying out numerical simulations is the modeling of the arterial compliance. Otherwise said, arterial walls deform under blood pressure, precisely they dilate during the systolic phase (when the heart is pumping) by storing elastic energy, and compress during the diastolic phase (when the heart inflates to be refilled by venous blood). This vessel displacement is dimensionally relevant (the increase of the arterial diameter during the systolic phase can reach the 10% of the

diameter at rest) and has crucial role in the circulatory functionality, as it contributes to keeping the blood pressure almost uniform from proximal (near the blood) to distal (near peripheries). From the physical viewpoint, wall compliance induces traveling pressure waves which are superimposed to the blood motion that is governed by the Navier-Stokes equations.

However, since blood flow interacts mechanically and chemically with vessel walls producing a complex fluid-structure interaction problem, which is impossible to simulate in its entirety, several reduced models have been developed which may give a reasonable approximation of averaged quantities, such as mean flow rate and pressure, in different sections of the cardiovascular system.

In this talk we address some mathematical issues arising from the modeling of the cardiovascular system through problems of different complexity and show their application to several problems of clinical interest.