GLOBAL OPTIMISATION OF NON-LINEAR INVERSE SCATTERING PROBLEMS

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ABSTRACT

This work is concerned with the inverse problem of the non ambiguous determination of the shape and composition of a generally noncircular 2D body from the way it scatters incident sound waves.

The inversion scheme consists in matching- for a particular frequency - the measured (or simulated) field to a theoretical/numerical estimation thereof and leads to a cost function (involving the unknown parameters of the object and which is non-linear in terms of these parameters) whose minimization enables the estimation of the parameters of the (unknown) object.

The ill-posed nature of such problem (non-unique and instable) is generally bypassed by adding to the cost function a weighted penalty term involving a-priori information on the object [1]- [3]. The solution then depends strongly on the choice of the weighting term.

Based on theoretical arguments (inverse crime and a high frequency approximation), recent work [4] demonstrates that the minima of the aforementioned cost function depend on the wavenumber of the probe radiation. This suggested the use of two frequencies to eliminate spurious solutions and has been successfully implemented to isolate the expected solution [5]. One drawback of this approach is that the choice of the two representative frequencies is ambiguous (a "certain" experiment is needed), the inverse problem has to be solved twice, each time for each particular frequency, and finally a post-treatment has to be set up to compare the positions of each local minimum with a given (arbitrary) tolerance. This is not compatible with real-time computations and still necessitates a priori information on the target. The present contribution tries to avoid these difficulties.

Our approach is to make a unique resolution for a particular cost function, which is the sum of each single frequency cost functions. The minimization task is then carried out in a global sense: only the global minimum is sought for and corresponds to the unknown parameter to reconstruct (Fig.1). Numerical simulations show the efficiency of the proposed method, even in the middle frequency range and when the inverse crime is not committed (Fig.2).



Fig.1: Multi-frequency cost function (square line '11') compared to various single frequency cost functions (continued lines '1'...'10'). The exact "local dimension of the target" in this particular scattering direction is 1.1.



Fig.2: reconstruction of both external and internal boundaries of a shell with a multifrequency cost function (N=10, for frequencies from 1000hz to 2000hz). Solid line: original target; dots: reconstructions at various scattering angles.

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