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Reflection Response of SH Transient Wave from a Finite Length Crack in Elastic Half-Space

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ABSTRACT

As the non-destructive evaluation is done for the health monitoring of structures, we can fundamentally obtain wave information by detecting the reflection energy from inclusions or material defects of impulsive waves. Some other similar engineering technologies are utilized over the industrial world(1)~(3). If we solve analytically such problem as a typical problem of the transient crack problem, the analysis ordinarily become a mixed boundary value problem, then we must solve the integral equation(4),(5). And as this numerical calculation could not be easily carried out, it was very difficult to represent bird's–eye responses over the full wave field. When we consider the propagation of transient waves, it is probable that the responses from the wave source and the reflection from observation objects independently generate. Therefore it becomes the more rational analysis that we analyze the transient problem for the elastic half-space and the wave reflection problem from the crack as independent two linear problems and superimpose them on real time and real space. Here, we consider the reflection problem from a finite length crack in the elastic half-space on the basis of the transient problem which the exact solution is given. The Cagniard method(6) gives the exact solution of an antiplane deformation problem which SH transient load acts on the free surface of a homogeneous half-space. The wave reflection problem from the micro crack in the half-space is solved by evaluation of the newly constructed wave source at the crack position and the Wiener-Hopf technique(7) and is reported by the author(8). Furthermore the transient problem of elastic layer can be solved by using the method of images(9). Here, above three transient problems are connected analytically, we are analyzing the reflection response of SH wave from the finite length crack in elastic half space and are giving the numerical results of the contour mapping representations.

If there is no crack in Fig.1, the equation of motion of the SH mode becomes as following

$$\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}=\frac{1}{c^2}\frac{\partial^2 w}{\partial t^2} \tag{1}$$

where, w is z direction displacement, c is phase velocity

$$c=\sqrt{\mu/\rho} \tag{2}$$

μ is the modulus of transverse elasticity and ρ is the mass density of the material.

The boundary condition on the free surface

$$\tau_{yz}\big|_{y=0}=Q\delta(x)H(t) \tag{3}$$

The initial conditions are

$$w\big|_{t=0}=\dot{w}\big|_{t=0}=0 \tag{4}$$

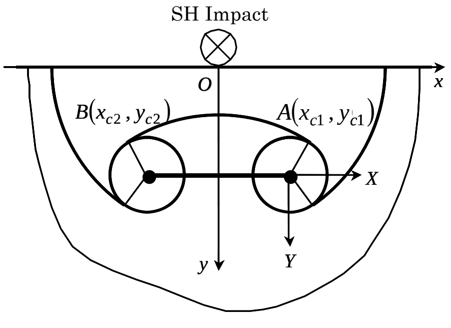

Fig.1 Reflection of SH transient waves from a finite-length crack

where, $\delta(x)$ is the Dirac's delta function and $H(t)$ is the Heaviside's step function.

Considering the boundary condition (3) and applying the Fourier Laplace double transforms to Eq.(1), we obtain the transformed solution. Furthermore, applying the Cagniard method to evaluate exactly the double inversion, we finally obtain the following exact solutions for the stress components and displacement.

$$\tau_{yz}=\frac{Q}{\pi}H(\tau-R)\frac{y\tau}{R^2\sqrt{\tau^2-R^2}} \tag{5}$$

$$\tau_{xz}=\frac{Q}{2\pi}H(\tau-R)\cdot\mathrm{Im}\left[\frac{\xi_+}{\sqrt{\xi_+^2+1}}\frac{\partial\xi_+}{\partial\tau}-\frac{\xi_-}{\sqrt{\xi_-^2+1}}\frac{\partial\xi_-}{\partial\tau}\right] \tag{6}$$

$$w=-\frac{Q}{\mu}\int_R^\tau \mathrm{Re}\left[\frac{1}{\sqrt{\xi_+^2(\sigma)+1}}\frac{\partial\xi_+(\sigma)}{\partial\sigma}-\frac{1}{\sqrt{\xi_-^2(\sigma)+1}}\frac{\partial\xi_-(\sigma)}{\partial\sigma}\right]d\sigma \tag{7}$$

and the strain energy U is

$$U=\frac{1}{2\mu}\left(\tau_{yz}^2+\tau_{xz}^2\right) \tag{8}$$

where, x, y, τ are given as the dimensionless quantities and

$$\xi_\pm=\frac{\pm y\sqrt{\tau^2-R^2}+ix\tau}{R^2},\ R=\sqrt{x^2+y^2} \tag{9),(10}$$

The reflection responses from the finite length crack were analyzed on the basis of these analytic solutions and were calculated numerically. Fig.2 is the contour mapping representations over the full wave field for stresses τ_{yz} , τ_{xz} , strain energy U and displacement w . Where, we set the A,B point coordinates up as (0.15,0.5) , (-0.35,0.5), respectively.

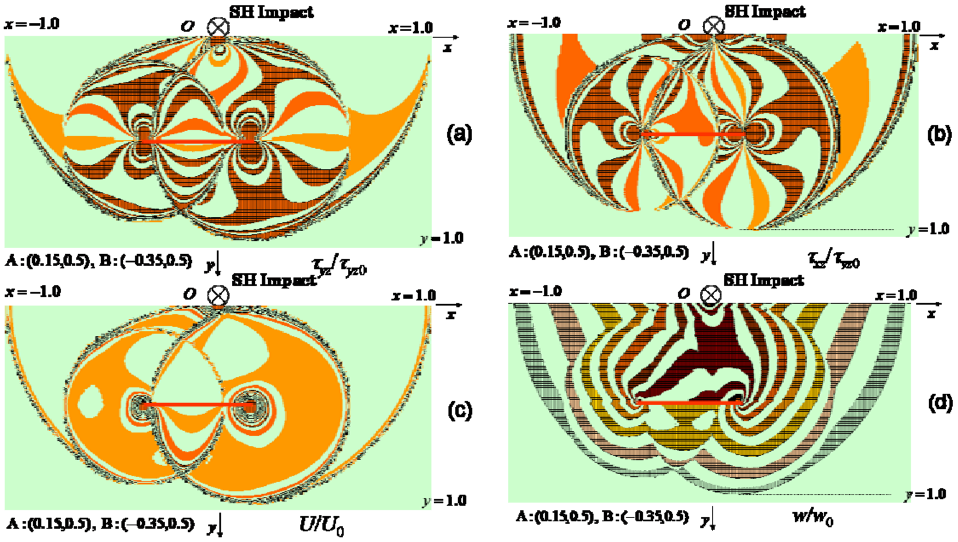

Fig.2 Transient reflection responses from a finite length crack with the crack tip coordinates A(0.15,0.5), B(-0.35,0.5).

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