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Acceleration strategies based on the reuse of Krylov subspaces in multiresolution problems with varying matrices

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ABSTRACT

This presentation deals with multiresolution problems, that is to say problems which require the resolution of more than one linear system. Many types of problems can be included in this framework: nonlinearity, evolution, optimization, event driven simulations... We focus on large problems where the matrices vary from one linear system to another, disabling us to use classical “multiple right hand sides” methods.

Our strategies are based on the use of non-overlapping domain decomposition methods [1,2,3] and Krylov iterative solvers [4] to solve the linear systems. We develop specific preconditioning [5] and projection [6] techniques to take advantage of the numerical information generated during previous systems to accelerate the following ones.

Basically the numerical information generated by a Krylov algorithm sums up to the basis of the Krylov subspaces. In a first approach, called Generalized Iterative Reuse of Krylov Subspaces [5], this information is reused to define an initialization and a precondioner which are optimal with respect to previous systems (that is to say the best which can be defined from previous resolutions). Taking advantage of the conjugation properties of Conjugate Gradient in the symmetric positive definite case or BiConjugate Gradient in more general cases, these initialization and preconditioner are very fast to compute, perfectly suited to parallel processing, and they lead to significant speedups (around 50% CPU time compared to non-accelerated domain decomposition method) on complex industrial structures (as for instance a flexible thrust bearing undergoing large transformations).

In a second approach, called Selective Reuse of Krylov Subspaces [6], the information is reused in an augmentation context (which, for the considered domain decomposition methods, can be regarded as an enriched coarse problem). Because augmentation with varying matrices relies on rather expensive computations, a selection of the information to reuse is made on spectral considerations: only the “best” Ritz vectors of previous systems are introduced in the current coarse problem. The results in term of conjugate gradient iterations can be really impressive (90% decrease) and, because of the higher cost of the involved operations (though most of them can be fully done in parallel), the CPU time gain is in

the same order of magnitude as previous method (around 50%). Initially defined on conjugate gradient, the method can be extended (with some difficulties) to GMRes for more general systems.

In this presentation the methods will be presented and assessed on different problems arising from various mechanical problems (nonlinear mechanics, optimization...) [7].

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