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A posteriori error analysis for nonstandard FEM

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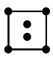
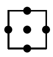
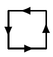
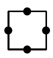
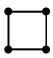
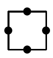
ABSTRACT

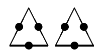
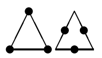
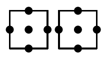
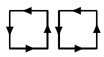
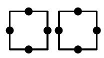
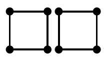
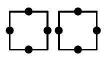
General strategies are discussed to derive a posteriori error estimates for conforming, mixed, and nonconforming finite element methods in energy norms which also cover discontinuous Galerkin schemes or Mortar finite elements for second order elliptic problems. The unifying approach provides reliable error estimates which can be shown to be efficient as well. One may say that *all* nonstandard schemes allow for error control, there is no finite element method known to the author where there is no error control. Surprisingly, there remains one type of residuals Res for different problems, such as, the Laplace problem, the Stokes problem, and Navier-Lamé problem. The main observation is that

$$\mathrm{Res}(v) := \int_\Omega g \cdot v dx + \int_{\cup\mathcal{E}} g_{\mathcal{E}} \cdot v ds \quad \text{for } v \in V$$

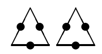
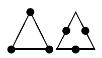
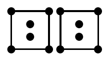
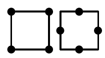
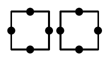
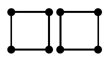
is the same for all those schemes. Some nonconforming elements are depicted in the following tables.

picture	name
	Crouzeix-Raviart
	Wilson
	Han
	NR (M)
	NR (A)
	CNR
	DSSY

NCFEM for Laplace

picture	name
	Crouzeix-Raviart
	Kouhia-Stenberg
	Han
	NR (M)
	NR (A)
	HMS
	CJY

NCFEM for Stokes

picture	name
	Brenner-Sung
	Kouhia-Stenberg
	Zhang
	Ming
	LLS
	HMS

NCFEM for Navier-Lamé

Method [Ref.]	$\hat{u}_T$	$\hat{p}_T$	c_1	c_2
Bassi-Rebay (1997)	$\{u_h\}$	$\{p_h\}$	-1	0
Brezzi et MMPR (1999)	$\{u_h\}$	$\{p_h\} - \alpha_r(\underline{[u_h]})$	-1	0
LDG of CS (1998)	$\{u_h\} - \beta \cdot \underline{[u_h]}$	$\{p_h\} + \beta[\sigma_h] - \alpha_j(\underline{[u_h]})$	-1	0
IP of DD (1976)	$\{u_h\}$	$\{Du_h\} - \alpha_j(\underline{[u_h]})$	-1	0
Bassi et RMPS (1997)	$\{u_h\}$	$\{Du_h\} - \alpha_r(\underline{[u_h]})$	-1	0
Baumann-Oden (1999)	$\{u_h\} + \nu_T \cdot \underline{[u_h]}$	$\{Du_h\}$	1	0
NIPG RWG (1999)	$\{u_h\} + \nu_T \cdot \underline{[u_h]}$	$\{Du_h\} - \alpha_j(\underline{[u_h]})$	1	0
Babuska-Zlamal (1973)	$(u_h\|_T)\|_{\partial T}$	$-\alpha_j(\underline{[u_h]})$	0	0
Brezzi et MM (2000)	$(u_h\|_T)\|_{\partial T}$	$-\alpha_r(\underline{[u_h]})$	0	0

Table 1: dG schemes and their specifications for LAPLACE

Method [Ref.]	$\hat{u}_{T,\sigma}$	$\hat{u}_{T,p}$	$\hat{\sigma}_T$	c_1	c_2
Bassi-Rebay (1997)	$\{u_h\}$	$\{u_h\}$	$\{\sigma_h\}$	-1	0
IP of SST (2003)	$\{u_h\}$	$\{u_h\}$	$\{Du_h\} - \alpha_j(\underline{[u_h]})$	-1	0
Bassi et RMMPR (1997)	$\{u_h\}$	$\{u_h\}$	$\{Du_h\} - \alpha_r(\underline{[u_h]})$	-1	0
NIPG of Toselli (2002)	$\{u_h\} + \underline{[u_h]} \cdot \nu_T$	$\{u_h\}$	$\{Du_h\} - \alpha_j(\underline{[u_h]})$	1	0
LDG of GKSS (2002)	$\{u_h\} + \underline{[u_h]} \cdot \beta$	$\{u_h\} + D_{11}[p_h]$ $+ D_{12}[u_h]$	$\{\sigma_h\} - [\sigma_h] \otimes \beta$ $- [p_h] \otimes \beta$ $- D_{12} \cdot [p_h] - \alpha_j(\underline{[u_h]})$	-1	1

Table 2: dG schemes and their specifications for STOKES

Method	$\hat{u}_{T,\varepsilon}$	$\hat{u}_{T,p}$	$\hat{\sigma}_T$	$\hat{p}_T$	c_1	c_2
IP of HL (2002)	$\{u_h\}$	$\{u_h\}$	$\{\varepsilon(u_h)\}$ $- \alpha_j(\underline{[u_h]})$	$-\lambda\{D \cdot u_h\}$ $+ \lambda\alpha_j([u_h])$	0	0
LDG of CKSS (2006)	$\{u_h\} + \underline{[u_h]} \cdot \beta$	$\{u_h\} + D_{12}[u_h]$ $+ D_{11}\underline{[p_h]}$	$\{\varepsilon_h\} - [\varepsilon_h] \otimes \beta$ $- \alpha_j(\underline{[u_h]})$	$\{p_h\} - D_{12} \cdot \underline{[p_h]}$	-1	1

Table 3: dG schemes and their specifications for NAVIER LAME

Several discontinuous Galerkin methods are analysed in the same unifying framework and the subsequent tables display a few examples. The unifying notation due to Cockburn and Shu is not recalled but employed to specify the methods which are not properly labelled in this abstract.

The conclusion of this presentation is sparsity in the mathematical research of a posteriori error control. The reduction is to two parts. (a) Analyze your new PDE in such a way that the error is equivalent to $\|\mathrm{Res}\|_*$ and analyze $V_h \subset \ker \mathrm{Res}$. (b) Design new a posteriori error estimates for $\|\mathrm{Res}\|_*$.

The presentation is partly based on joint work [1–4] with Jun Hu, Antonio Orlando, Max Jensen, and Thirupathi Gudi.

REFERENCES

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[4] C. Carstensen, Thirupathi Gudi and Max Jensen, A unifying theory of a posteriori error control for discontinuous Galerkin FEM, submitted to Numerische Mathematik.