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FAST ALGEBRAIC BOUNDARY INTEGRAL SOLVER FOR FAR FIELD ACOUSTIC UNBOUNDED PROBLEM USING ADAPTATIVE CROSS APPROXIMATION MATRIX

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ABSTRACT

The main problem in applying boundary element methods (BEM) in computational far field acoustic simulation is related to the large memory requirements of the matrix ($\mathcal{O}(n^2)$) and the complexity of the matrix-vector product.

Modern numerical methods for the solution of BEM equation provide an approximation $\tilde{x}$ to the solution x with almost linear complexity by solving a perturbed linear system where the associated matrix is easier to handle. The starting point of these modern numerical methods is the fast multipole method (FMM) published by Rokhlin ([1]) which focussed on multiplication of BEM matrices by a vector. The panel clustering method proposed by Hackbusch and Nowak was designed for the fast matrix-vector product for collocation matrices and general kernels κ which allow a *degenerate* approximation $\tilde{\kappa}$:

$$\kappa(x, y) \approx \tilde{\kappa} = \sum_{i=1}^{k} u_i(x)v_i(y), x \in D_1, y \in D_2$$

on a pair of domains D_1 and D_2 satisfying $\eta.\text{dist}(D_1, D_2) > \text{diam}(D_2)$ where $\eta > 0$ is a parameter and k a small number compared to the dimension N of the problem.

A difficulty of the FMM and the panel clustering method is that functions u_i and v_i have to be known explicitly. So these methods are strongly tied to the kernel operator and both methods need a complete recoding of the matrix–vector product including computation of the coefficients.

What we propose here, using a Brakhage-Werner formulation for the integral formulation, is an algebraic approach of the kernel approximation based on the Adaptive Cross Approximation (ACA) method initially published by Bebendorf [3] for asymptotically smooth kernel operators, and improved by Grasedyck ([4]). A compression using a cheap singular value decomposition (SVD) ([4]) improves the performance of the compression. Thanks to the algebraic approach, ACA method can be used as a black box, computing a low–rank approximation of appropriate matrix blocks, independent on the kernel operator.

A first result is provided on the spherical academic test case and compared with the semi-analytic solution. Results on industrial cases (an aircraft nacelle with 6745 degrees of Freedom (DoF)), compared

with the classical full plain BEM matrix approach(Fig. 1), will allow to evaluate the ACA compression with or without the SVD recompression (Fig. 2).

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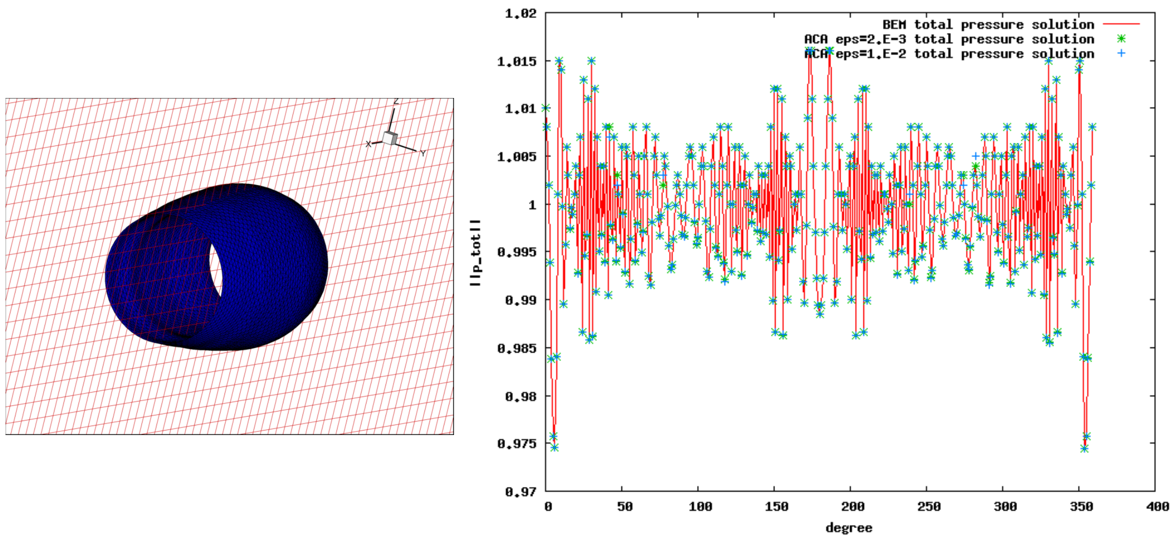

Figure 1: Nacelle case with incident plane wave (6000Hz) and computation of external pressure on a 3m radius circle with axis $< Oy >$

Plain Matrix		Time [Sec]	Storage [Kb/Dof]		
-	-	2129	105.4	-	
Aca Tol. err.	SVD tol. err.	Time [Sec]	Storage [Kb/DoF]	ratio $\mathcal{H}$/Plain Matrix	error L2 with BEM
1.E-3	-	1548	23.3	22.1%	1.03E-3
2.E-3	-	1440	21.4	20.3%	1.04E-3
2.E-3	2.E-3	1424	16.3	15.45%	1.76E-3
2.E-3	1.E-2	1424	12.6	11.9%	2.07E-3
2.E-3	2.E-2	1424	11.2	10.6%	4.11E-3
1.E-2	-	1104	16.9	16%	2.76E-3

Figure 2: Performance of ACA compression on the nacelle problem (6000Hz)