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REDUCED ORDER MODELING OF THERMAL CONVECTION UNDER A MAGNETIC FIELD

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ABSTRACT

Control methods for fluid dynamics have attracted substantial interest in recent years due to their wide ranging applications [1]. Since real-time flow control requires large amounts of computer-time and memory, it is very important to have efficient computational schemes. Among such schemes, reduced order models play an important role. This requires a suitable representation and parametrization of the underlying flow field. Karhunen-Loeve (KL) decomposition technique is used to construct a set of basis functions from a flow database providing an optimal representation of the flow field in energy norm [2]. These basis functions are subsequently used to construct a reduced order model of the flow dynamics [3].

Thermal convection under a magnetic field, magneto-convection, is of interest in some industrial processes such as molten metal casting and crystallization, where the control of the convective flow by a magnetic field may influence the quality of the produced material [4], as well as in the laboratory, where the imposed external magnetic field is exploited as a convenient control parameter with which the dynamics of the convective flow could be changed [5]. Computationally, an electrically conducting layer between rigid plates heated from below in the presence of a magnetic field is the simplest geometry to study the interaction between thermal convection and externally imposed magnetic field [6].

In this work, we propose a KL based reduction scheme in order to formulate a reduced order model of a magneto-convective flow. This model is obtained from a KL based model of Rayleigh-Benard (RB) convection by adding the effect of a magnetic field. The scheme consists of extending the KL basis spanning a RB convection flow field to construct a reduced order model for a magneto-convective flow under Galerkin projection. For doing this, we exploit the quasi-steady relationship between the velocity and the induced magnetic field obtained under the low magnetic Prandtl number limit [7]. Besides typically having low thermal Prandtl number, the liquid metals or melts are characterized by this limit. This corresponds to the physical situation that while the induced magnetic field hardly interacts with convection, the ambient homogeneous magnetic field affects the convective motions. As a consequence, the induced magnetic

field can be viewed as a slaved variable, prescribed by the velocity field.

For the purpose of constructing KL basis functions, a database representing RB convection flow field is numerically generated by integrating Boussinesq equations within a computational domain of a horizontally periodic convective box between upper and lower rigid plates. The numerical technique is based on a spectral element method developed earlier to simulate natural thermal convection [8]. Its main features are the variational (weak) formulation in the wall-to-wall direction and the use of rescaled Legendre-Lagrangian polynomial interpolants in expanding the flow variables except the pressure for which a modal expansion in terms of lower order polynomials is used to avoid the complicated staggered grid approach.

The generated KL basis is used to reduce the Boussinesq equations to a system of amplitude equations by a Galerkin projection [9]. In the generation of the KL basis, flow symmetries are fully exploited in order to pass as much flow character on to the basis functions. The resulting amplitude equations are truncated based on the physical character attached to as well as the amount of flow energy carried along each KL basis functions at the reference flow parameter values. The Lorentz force, resulting from the interaction of convection with the magnetic field, is then included in the amplitude equations. The further reduced and modified amplitude equations are then numerically integrated to explore the dynamics in a range of flow parameter values. The effect of a magnetic field in such a way to inhibit the convective motions is shown to be exhibited in the solutions of the reduced model.

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