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A THERMOMECHANICAL REFINED SINUS FINITE ELEMENT INCLUDING TRANSVERSE NORMAL STRESS EFFECTS IN MULTILAYERED BEAMS

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ABSTRACT

Composite and sandwich structures are widely used in industrial field due to their excellent mechanical properties. In this context, they can be submitted to severe conditions which imply to take into account thermal effects. In fact, they can play an important role on the behaviour of structures in services, which leads to evaluate precisely their influence on stresses, particularly at the interface of layers.

The aim of this paper is to construct a finite element for analyzing laminated beams including thermomechanical effects in elasticity for small displacements. In particular, transverse normal stress effects are taken into account.

From the litterature, various theories developped in mechanics for composite or sandwich structures were extended to include thermal effects. They can be classified as:

- the Equivalent Single Layer (ESL): the number of unknowns are independent of the number of layers, but the shear stress continuity at the interfaces of layer are often violated. We can distinguish the classical laminate theory, the first order shear deformation theory, and higher order theories [1], which analyse thermal stresses for beams and plates.
- the Discrete Layer theory or layer-wise approach (DLT): this theory aims at overcoming the restriction of the ESL about the discontinuity of in-plane displacement at the interface layers. In this framework, Kapuria used a zigzag theory for displacements with continuity of shear stresses in the thickness for beams and plates [2]. See also [3]. Unfortunately, the cost increases with the number of plies.

Thus, we propose a new thermomechanical finite element for rectangular laminated beam analysis, so as to have a low cost tool, efficient and simple to use. In fact, our approach is associated with the

ESL theory based on a sinus distribution ([4]) with layer refinement ([5,6]). This particular kinematic is enriched by three unknowns per layer. It accounts for the interlaminar continuity conditions on the interfaces between the layers (transverse shear stress and displacement), and the boundary conditions on the upper and lower surfaces of the beam, owing to the Heaviside function and the double superposition hypothesis from [7]. Therefore, this approach takes into account physical meaning.

From this requirement, this model is composed of 3 usual generalized displacements (2 displacements and 1 rotation), two components of quadratic transverse displacement, and only one more unknown. So, It is important to notice that the number of unknowns is independent from the number of layers.
In the framework of a conforming FE approach, a new three-noded coupling beam F.E. is carried out using Lagrange (axial displacement and rotation of the section) and Hermite (transverse displacement) interpolations. So, this element is totally free of shear locking, without any classical numerical pathologies, and the present kinematics allows to avoid the use of shear correction factors for laminates.

Thermomechanical tests for thin and thick beams with various stacking sequences are presented in order to evaluate the capability of this new finite element to give accurate results with respect to elasticity solutions [8]. Both convergence velocity and accuracy are discussed and this new finite element yields very satisfactory results at a low computational cost. In particular, the transverse stress computed from constitutive relation is well estimated with regards to classical equivalent single layer models. Moreover, these examples put the emphasis on the necessity to take into account the transverse normal stress effect in the thermomechanical coupling as in [9].

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