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A Geometrically Nonlinear Finite Shell Element for the Analysis of Piezoelectric Smart Structures.

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ABSTRACT

In this contribution a finite element formulation to analyze piezoelectric shell problems is presented. In recent years several new shell elements have been proposed, see e.g. [1]-[4]. Smart materials and structures play an important role for sensor and actuator applications. To predict the material and system behavior the simulation of such structures is essential. A reliable simulation may provide an easier, faster and cheaper development of these structures. For this reason we provide a geometrically nonlinear most accurate finite element formulation to analyze piezoelectric shell structures. The formulation is based on a mixed field variational principle of Hu-Washizu with six independent fields. These are displacements $\mathbf{u}$, electric potential $\Delta\varphi$, strains $\mathbf{E}$ and electric field $\vec{\mathbf{E}}$ summarized in the vector $\hat{\varepsilon}(\mathbf{E}, \vec{\mathbf{E}})$, stresses $\mathbf{S}$ and dielectric displacements $\vec{\mathbf{D}}$ summarized in the vector $\hat{\sigma}(\mathbf{S}, \vec{\mathbf{D}})$. The variational formulation reads

$$\Pi(\mathbf{u}, \Delta\varphi, \mathbf{E}, \vec{\mathbf{E}}, \mathbf{S}, \vec{\mathbf{D}}) = \int_{\mathcal{B}} W_s(\hat{\varepsilon})\, dV + \int_{\mathcal{B}} \hat{\boldsymbol{\sigma}}^T (\varepsilon_g - \hat{\varepsilon})\, dV - \Pi^{ext}(\mathbf{u})$$

W_s is the stored energy function, which is defined as $W_s = \frac{1}{2}\hat{\varepsilon}^T \bar{\mathbb{C}} \hat{\varepsilon}$. ε_g is the vector of the geometric strains $\mathbf{E}_g$ and the geometric electric field $\vec{\mathbf{E}}_g$. The mixed formulation allows an interpolation of the strains and the electric field through the shell thickness. Regarding the constitutive law no simplifications are assumed. According to a linear theory of piezoelectricity, the constitutive relations are presented in the form

$$\begin{bmatrix} \mathbf{S} \\ -\vec{\mathbf{D}} \end{bmatrix} = \underbrace{\begin{bmatrix} \mathbb{C} & -\mathbb{e}^T \\ -\mathbb{e} & -\epsilon \end{bmatrix}}_{\bar{\mathbb{C}}} \begin{bmatrix} \mathbf{E} \\ \vec{\mathbf{E}} \end{bmatrix}$$

where $\mathbb{C}$ is the three dimensional elasticity matrix, $\mathbb{e}$ the piezoelectric matrix and ϵ the matrix of the permittivity. The 3D material law is an essential feature of this work, which allows in the future the consideration of material nonlinearities.

We propose a four node element with bilinear shape functions, which models the shell with a reference

surface. The element has seven degrees of freedom, three displacements, three rotations, and one electrical degree of freedom, which is the difference of the electrical potential in thickness direction of the shell. By means of the isoparametric concept the displacements, the electric potential, and the geometry are approximated with the same shape functions.
To fulfill the normal zero stress condition and the normal zero dielectric displacement condition, the stress and the dielectric displacement in thickness direction are enforced to be zero by the independent stress resultants and the independent dielectric displacement vector.

The developed mixed hybrid shell element fulfills the important patch tests, the in-plane, bending and shear test, which have been adopted for coupled field problems. In order to prove the applicability of the piezoelectric element some numerical examples are presented. Here, a cylinder used for sensor applications is investigated, see Fig. 1. The cylinder with the height $H = 25.4$ mm, the outer diameter $d_a = 19.1$ mm and the inner diameter $d_i = 17.3$ mm is subjected to the linear distributed load F. The material parameters are given as Young's modulus $E = 67.4 \cdot 10^9 \frac{N}{m^2}$, Poisson's ratio $\nu = 0.31$, $\mathbb{e}_{13} = -9.30032 \frac{C}{m^2}$, $\mathbb{e}_{33} = 20.3638 \frac{C}{m^2}$, $\mathbb{e}_{15} = 14.5749 \frac{C}{m^2}$ and $\epsilon_{11} = \epsilon_{33} = 15.31742 \cdot 10^{-9} \frac{C^2}{Nm^2}$. The system is clamped at the lower edge. The load induces a displacement and an electric potential. The geometric nonlinear effect, which arise due to the theoretical load, is depicted in Fig. 1.

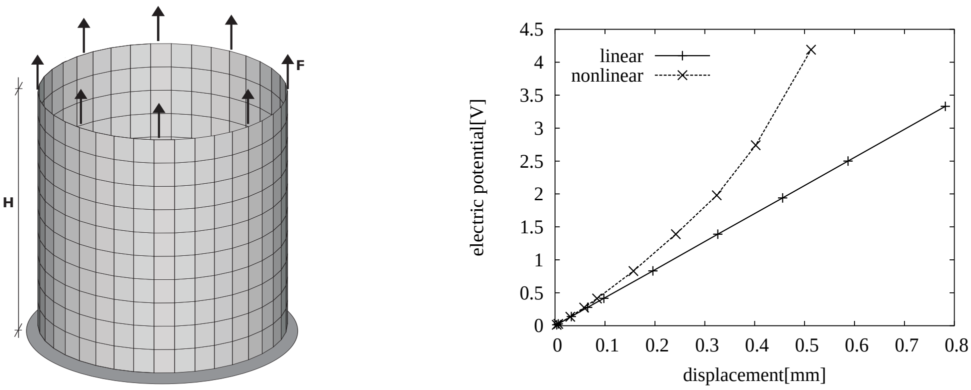

Figure 1: Cylindrical shell, geometry and diagram of the correlation between axial displacement and electric potential

Furthermore some simulations of practical devices are shown to underline the practical use of the element. An important feature is thereby the analysis under the consideration of geometrical nonlinearity.

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