8th. World Congress on Computational Mechanics (WCCM8)
5th. European Congress on Computational Methods in Applied Sciences and Engineering (ECCOMAS 2008)
June 30 – July 5, 2008
Venice, Italy

A BONE REMODELLING MODEL FOR APPARENT DENSITY AND TRABECULAR ARCHITECTURE

***Paulo R. Fernandes[1], Pedro G. Coelho[2], José M. Guedes[3], Helder C. Rodrigues[4]**

[1] IDMEC/IST, TU Lisbon
Lisbon, Portugal
prfernan@dem.ist.utl.pt

[2] FCT-UNL
Caparica, Portugal
pgc@fct.unl.pt

[3] IDMEC/IST, TU Lisbon
Lisbon, Portugal
jmguedes@ist.utl.pt

[4] IDMEC/IST, TU Lisbon
Lisbon, Portugal
hcr@ist.utl.pt

Key Words: *Bone remodelling, Hierarchical Structure, Bone Anisotropy, Porosity*

ABSTRACT

Bone is a hierarchical structural material, since several organizational levels can be identified from the macroscale to the nanoscale. The two top levels corresponding to the entire bone and trabecular structure respectively show a suitable distribution of physical properties, such as bone density and corresponding mechanical properties, to achieve the functional requirements of bone tissue.

In this work a three-dimensional model for bone remodelling is developed taking into account the hierarchical structure of bone. The process of bone tissue adaptation is mathematically described with respect to functional demands, both mechanical and biological factors, to obtain the bone apparent density distribution (at the macroscale) and the trabecular structure (at the microscale). At global scale bone is assumed as a continuum material characterized by equivalent (homogenized) mechanical properties. At local scale a locally periodic cellular material model approaches bone trabecular anisotropy in terms of mechanical properties. For each scale there is a material distribution problem governed by density based design variables which at the global level can be identified with bone relative density [1].

The law of bone remodelling assumes that bone adapts to functional demands in order to satisfy a multi-criteria objective base on structural stiffness (maximization) and metabolic cost k of bone formation (minimization). Assuming such a multi-criteria and setting a weighted set of P loads, characterizing the loading environment applied to bone, the bone distribution on the micro level is given by the remodelling rate equation,

$$\frac{\partial \mu}{\partial t} = \frac{\partial E^H_{ijkl}(\mu)}{\partial \mu} \sum_{r=1}^{P} \left[\alpha^r \varepsilon_{ij}(\mathbf{u}^r) \varepsilon_{kl}(\mathbf{u}^r) \right] - k \quad (1)$$

where μ is the microscale density (trabecular level), $\varepsilon(\mathbf{u}^r)$ is the strain tensor for the displacement field $\mathbf{u}^r$ solution of the equilibrium equation for the global problem and the r^{th} load, and E^H_{ijkl} is the homogenized properties for the trabecular bone. The macro-density ρ is obtained by the micro-density μ through, $\rho(x) = \int_Y \mu(x,y)\,\mathrm{d}Y, \quad \forall x \in \Omega$ where Ω corresponds to the whole bone domain and Y is the micro-cell domain.

Three-dimensional examples are presented to illustrate the model. The density distribution for a femur is shown in figure 1a and figure 1b illustrates the microstructures of elements selected from different anatomic regions. The model is able to provide a density distribution that fairly approximates the real femur bone at macroscale. At microscale the microstructures give a good mechanically characterization of the local microstructure of trabecular bone with the respective anisotropic properties. Previous results show that some microstructures with maximum mechanical efficiency do not match porosity levels observed in real bone [2]. In this work biological requirements, such as porosity constraints, are carefully considered for a better representation of bone tissue adaptation. The inclusion of porosity constraints produces a model able to obtain microstructures that are mechanically acceptable and simultaneously satisfy biological requirements. Figure 2 shows compact bone obtained with the model. The porosity and the material symmetry obtained compares fairly with experimental data [3].

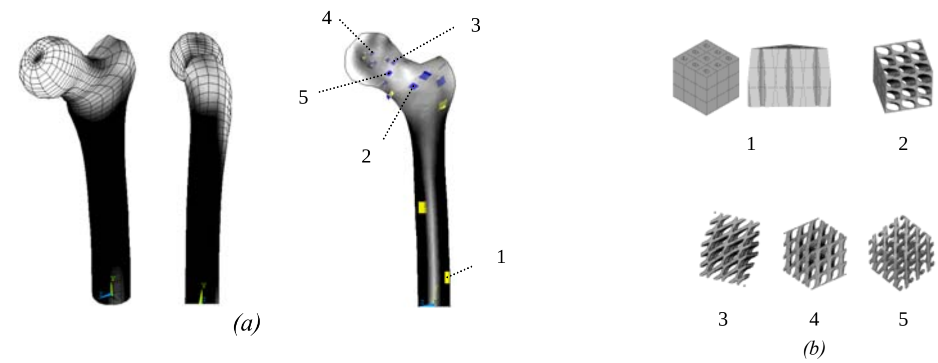

Figure 1 - (a) Density distribution on femur. (b) Microstructures for selected regions of bone.

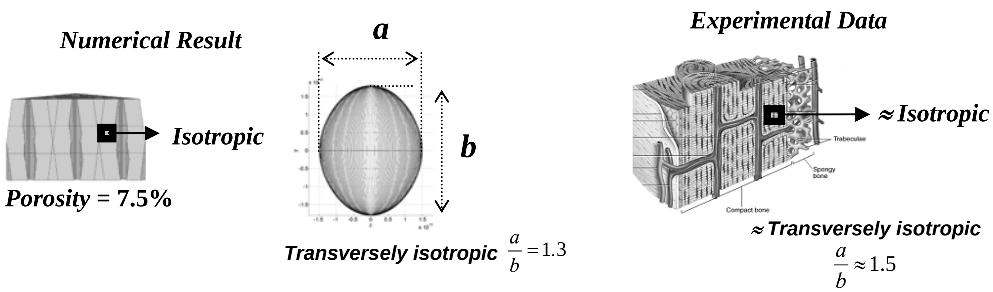

Figure 2 - Porosity of compact bone.

ACKNOWLEDGEMENT

This work was supported by FCT through the project: PTDC/EME-PME/71436/2006.

REFERENCES

[1] P. Coelho, P. Fernandes, J. Guedes and H. Rodrigues, “A hierarchical model for concurrent material and topology optimization of three-dimensional structures”, Int. J. Struct and Multidisc Opt, (2008), in press.

[2] O. Sigmund, “On the optimality of bone structure”, Proc. IUTAM Symp. Synth. in Bio Solid Mech.. Copenhagen, Denmark, 1999

[3] R. Martin, D. Burr, N. Sharkey, “Skeletal Tissue Mechanics”, Springer-Verlag, NY (1998).