

## ENERGY CONSISTENT INTEGRATORS FOR DISSIPATIVE MULTIBODY SYSTEMS

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### ABSTRACT

We present a rotationless formulation for multibody systems which has been shown to be especially well-suited for the design of energy-momentum conserving integration schemes [1, 2]. Energy-momentum schemes facilitate a stable numerical integration of differential algebraic equations (DAEs) with index three. The rotationless formulation of rigid bodies relies on redundant coordinates which coincide with the components of the rotation matrix. The constraint of rigidity is explicitly enforced by means of six independent constraints for enforcing the orthonormality of a body fixed director frame. The underlying DAEs facilitate the incorporation of additional 'external' constraints which account for the interconnection of rigid bodies in a multibody framework.

Although the rotationless formulation does not make use of rotational parameters, rotations are often required to model specific multibody systems. For example, joint-angles and associated joint-torques are common modeling features. To incorporate rotations into our rotationless formulation we have previously developed in [1, 2] a coordinate augmentation technique. This approach represents an extension of the original rotationless formulation and thus preserves all the advantageous algorithmic conservations properties of the original scheme. In particular, for conservative multibody systems with symmetry, the time-stepping scheme inherits conservation of both the momentum maps (linear and angular momentum) and the total energy from the underlying continuous system.

Of course, friction effects play a major role in real world applications. Our goal is to develop an energy consistent integration method for nonconservative multibody system. In particular, in the limit case of vanishing friction the original conserving method should be recovered. An energy consistent method should capture the dissipative properties correctly, independent of the time step. Standard integrators do not possess this property in general, see [3]. In the present work we will focus on the incorporation of joint-friction by making use of the aforementioned coordinate augmentation technique. Representative numerical examples will deal with open-loop as well as closed-loop multibody systems.

### REFERENCES

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