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COUPLED THERMOELASTICITY OF FG THIN SHELLS: EFFECT OF TEMPERATURE FIELD ACROSS THE THICKNESS

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ABSTRACT

The effect of temperature field for linear and non-linear polynomial for the temperature distribution across the shell thickness in the problem of coupled thermoelastic response of a functionally graded (FG) thin shell is presented. A shell classical approximation is considered. The coupled energy equation based on the assumption of Lord and Shulman is considered. The coupled thermoelastic and the temperature equations are simultaneously solved for a functionally graded axisymmetric thin shell subjected to thermal shock load. The shell is graded through the thickness assuming a volume fraction of metal and ceramic, using a power law distribution. Including the thermo-mechanical coupling and rotary inertia, a Galerkin finite element formulation in space domain and the Laplace transform in time domain is used to formulate the problem. The results are validated with the known data in the literature.

1. Theory

The coupled thermoelasticity theory deals with the situation which the characteristic times of structural and thermal disturbances are of comparable magnitudes, and the equations of motion of a structure are coupled with the energy equation. In such a case the solution of the coupled system of equations provides the stress and temperature fields in the shell [1]. A first-order shear deformation shell theory is considered, including the rotary inertia term. The present article discusses two different polynomials for the temperature distribution across the shell thickness in the coupled thermo-elasticity problem of a functionally graded thin shell under thermal shock input. The effect of these two temperature fields across the shell thickness is discussed. A second-order polynomial (quadratic form) for the temperature distribution across the shell thickness is considered, and the results are compared to the linear temperature distribution. The temperature and stress fields are expressed as a function of cylindrical coordinates (x, θ , z) and time t. For the linear approximation, the temperature distribution is assumed as

$$\Delta T_1(x,\theta,z,t) = T_0(x,\theta,t) + zT_1(x,\theta,t) \quad (1)$$

here T_0 and T_1 are both unknowns. For the quadratic distribution we assume a temperature distribution as

$$\Delta T_2(x,\theta,z,t) = T_0(x,\theta,t) + zT_1(x,\theta,t) + \frac{1}{2}z^2T_2(x,\theta,t) \quad (2)$$

where T_0, T_1, and T_2 are some unknown functions and must be obtained through the coupled system of equations [5]. The results are validated with the known data in the literature [1], [2], [3], [4], and [5].

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2. Results and Discussion

A clamped FG thin shell of length 500 *mm*, diameter 200 *mm*, and wall thickness 2 *mm* under the inside impulsive thermal shock is considered. The functionally graded shell is assumed to be at the initial temperature 298.15 K, with the material properties of Titanium (Ti-6Al-4V) and Zirconia (ZrO2) [1]. The shell is ceramic-rich in inside and metal-rich at the outside surfaces, respectively. Thermal shock is applied to the inside surface. The equation of thermal shock is [1]

$$T(t) = 2201.85\, aTe^{-13100bt} + 298.15\ ^{o}K \qquad (3)$$

where a and b are selected such that the impulsive shock period becomes 10^{-6} *sec*. The coefficient of thermal convection of the ceramic-rich inside surface is (h_{in}=10,000 W/m^{2}K) and that of the metal rich outside surface is (h_{out}=200 W/m^{2}K). The thermal boundary conditions at the ends of the shell are assumed insulated. For the functionally graded shell the power law index is assumed to be k = 3. The Lord-Shulman thermal relaxation time t_0 of thin shell is assumed 1.5 * 10^{-6}. Figure (1) illustrates the axial force of the shell middle length versus time. It is observed that the axial force of parabolic temperature field is smaller than that of linear filed. The axial stress distribution along the shell thickness of FG thin shell is shown in Fig. (2). This figure clearly illustrates that the axial stress of parabolic temperature field is almost larger than that of linear field.

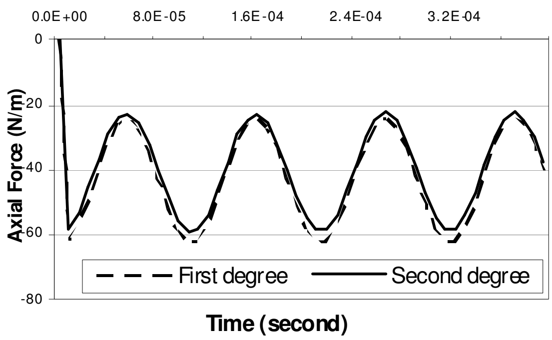

Figure 1: Axial force vs time at the middle length of the FG thin shell

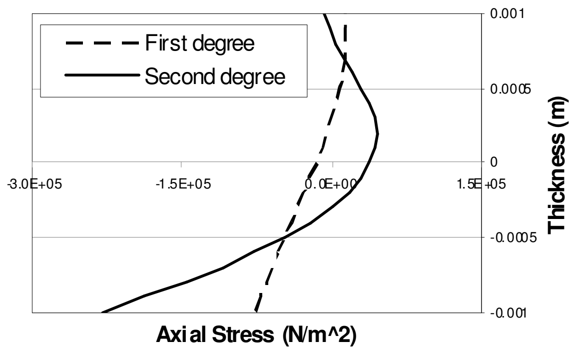

Figure 2: Axial stress distributions along the thickness at the mid-length of the FG thin shell at t=200 μ sec.

3. Conclusions

The effect of temperature field for linear and non-linear polynomial for the temperature distribution across the shell thickness is studied in a functionally graded thin shell. Linear distribution for temperature does not represent the actual effects of the temperature increase due to the thermal shock loading at the inside surface. The quadratic distribution, however, shows a fluctuant behavior which may be not suitable for analysis. Further research should be performed as well as superimposing a linear interpolation across the shell thickness.

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