8th. World Congress on Computational Mechanics (WCCM8)
5th. European Congress on Computational Methods in Applied Sciences and Engineering (ECCOMAS 2008)
June 30 – July 5, 2008
Venice, Italy

Multiscale Equivalent Aggregating Discontinuities: Circumventing Loss of Ellipticity

***Jeong-Hoon Song[1], Stefan Loehnert[2] and Ted Belytschko[3]**

[1] Northwestern University
2145 Sheridan Road
Evanston, IL 60208, U.S.A
j-song2@northwestern.edu

[2] Leibniz Universitaet Hannover
Appelstr. 9A
30167 Hannover, Germany
loehnert@ibnm.uni-hannover.de

[3] Northwestern University
2145 Sheridan Road
Evanston, IL 60208, U.S.A
tedbelytschko@northwestern.edu

Key Words: *multiscale, fracture, discontinuity, extended finite element method.*

ABSTRACT

New multiscale method for the analysis of failure that invokes unit cells to obtain the subscale response is described. This method, called Multiscale Equivalent Aggregating Discontinuities (MEAD) [1], is based on the concept of "perforated" unit cells, which excludes subdomains that are unstable, i.e. exhibit loss of material stability. By means of this concept, it is possible to compute an equivalent discontinuity at the finer scale, including both the direction of the discontinuity and the magnitude of the jump. These variables are then passed to the coarse scale model along with the stress in the unit cell. The discontinuity variables at the coarser scale are invoked by injecting the discontinuity by the extended finite element method (XFEM) [2, 3] procedure.

1. Multiscale Equivalent Aggregating Discontinuities Method

Multiscale methods such as FE^2 [4] are widely used because they enable the constitutive equations at the macroscale to be computed on-the-fly from microscale models. However, the classical FE^2 method is not applicable to failure problems. In this study, we have developed a way to apply FE^2 to failure problems.

In the MEAD method, the coarse scale model is linked to unit cells with fine scale details. Generally, these unit cells are only linked to “hot” spots, where a preliminary computer simulation indicates that loss of a material stability, i.e. failure is likely. At each quadrature point, the coarse scale model passes a measure of the deformation to the unit cell. The deformation gradient, $\mathcal{F}$, is passed so that the method is applicable to large deformations and material nonlinearities. The unit cell boundary condition is then prescribed by the following displacement

$$\mathbf{u}^m = \left(\mathcal{F}^M - \mathbf{I}\right)\cdot \mathbf{X}^m \qquad (1)$$

where $\mathbf{u}$ is the displacement, $\mathbf{I}$ is the identity tensor and $\mathbf{X}$ is the material coordinate. We use a superscript m and M to denote variables associated with the microscale (fine scale) and the macroscale (coarse scale or coarse-grained) model, respectively.

An equivalent discontinuity is extracted from the deformation of the unit cell which can be passed to the coarser scale. This is accomplished by using a form of Hill’s theorem; see, Zohdi and Wriggers [5, p. 59].

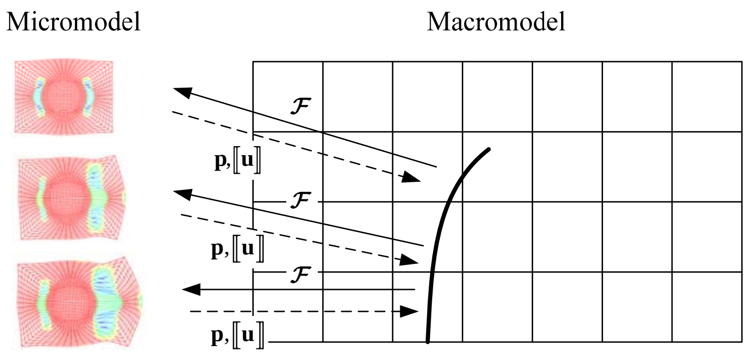

Figure 1. Schematic of MEAD method; the macro model provides deformation gradient $\mathcal{F}$ to the unit cells and stress $\mathbf{P}$ and equivalent discontinuities $[\![\mathbf{u}]\!]$ are passed back to the macroscale.

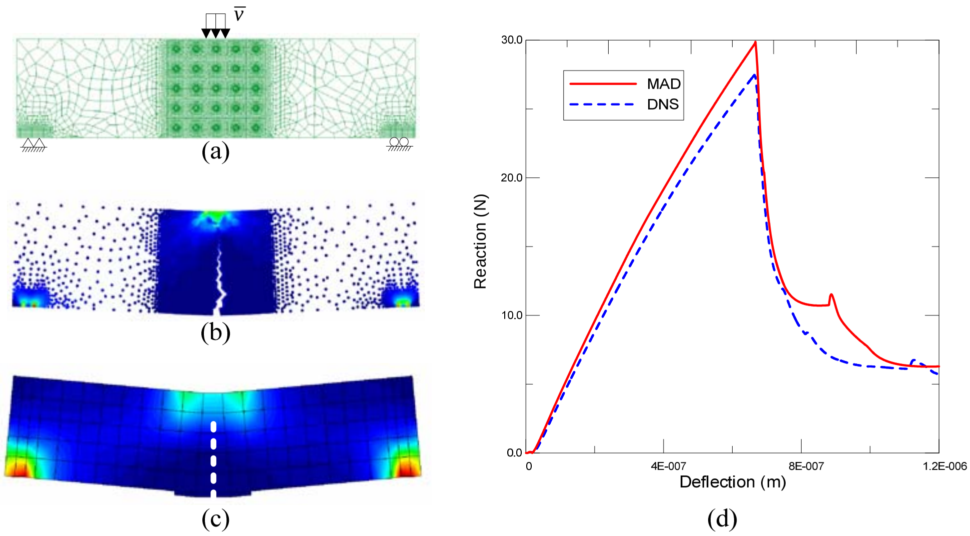

Figure 2. Comparison of the results from direct numerical simulation (DNS) and MEAD: (a) the undeformed model for DNS, (b) solution from the DNS, (c) solution from the MEAD, and (d) comparison of the force deflection curves.

REFERENCES

[1] T. Belytschko, S. Loehnert and J.-H. Song, “Multiscale Aggregating Discontinuities: A Method for Circumventing Loss of Material Stability”, *International Journal for Numerical Methods in Engineering*, DOI: 10.1002/nme.2156 , in earlyview.

[2] N. Moes and J. Dolbow and T. Belytschko, “A finite element method for crack growth without remeshing”, *International Journal for Numerical Methods in Engineerin*, **46**: 131-150, 1999.

[3] J.-H. Song, P. M. A. Areias, and T. Belytschko, “A method for dynamic crack and shear band propagation with phantom nodes”, *International Journal for Numerical Methods in Engineering*, **67**: 868-893, 2006.

[4] F. Feyel and J. L. Chaboche, “FE^2 multiscale approach for modeling the elastoviscoplastic behaviour of long fibre SiC/Ti composite materials”, *Computer Methods in Applied Mechanics and Engineering*, **183**:309-330, 2000.

[5] T. I. Zohdi and P. Wriggers, “*Introduction to Computational Micromechanics*”, Lecture notes in applied and computational mechanics, v. 20, Springer, Berlin, 2005.