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HIGH ORDER EXTENDED FINITE ELEMENT METHOD : INFLUENCE OF THE GEOMETRICAL REPRESENTATION

*** K. Dréau[1], N. Chevaugeon[1] and N. Moës[1]**

[1] Institut de Recherche en Génie Civil et Mécanique, UMR CNRS 6183, École Centrale de Nantes, B.P. 92101, 44321 Nantes Cedex 3, France
{kristell.dreau, nicolas.chevaugeon, nicolas.moes}@ec-nantes.fr

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ABSTRACT

The Extended Finite Element Method (X-FEM) is a good alternative to the classical finite element method for solving problems with complex geometries. Within the classical finite element method, the mesh is required to conform to physical surfaces. Discontinuities such as holes, cracks and material interfaces can't cross mesh elements. Moreover, local refinements close to discontinuities and mesh modification to track the geometrical and topological changes in crack propagation problems for example, can be difficult. Also, robust methods to transfer the solution to the new mesh are needed.

Within the X-FEM, surfaces that are not represented explicitly by mesh boundaries can be implicitly represented by the iso-values of a level-set function. This is particularly useful for moving interfaces problems such as crack surfaces in crack propagation analysis [1]. The finite element approximation is enriched by additional functions through the notion of partition of unity, to represent, for example, discontinuities at interfaces [2] or asymptotic expansions terms near crack tip [1] to improve convergence rate. The enrichment functions are usually defined with the help of the level-set functions to access the distance to the interface at any given point. Although interfaces do not have to be meshed, the correct integration of the element stiffness matrix, for elements enriched by a discontinuous function along an interface, need to be done carefully. The elements are split at the integration level along the iso-zero value, and regular integration is done separately on each side of the integration partition. Difficulties to represent the surface in the classical finite element method are, in the context of X-FEM, partly shifted on the integration procedure. Nevertheless, X-FEM gives good results with linear elements and linear level-set in elements : optimal rate of convergence is achieved with curved geometries. With shape functions of higher order, if the error is reduced, the optimal rate of convergence can't be achieved without improving the geometrical representation. The problem is of course related to the well known fact that, when increasing the order of the approximation field, the order of representation of the geometry must be increase accordingly to get optimal rate of convergence, in the classical finite element method. Iso-parametric elements have an energy error norm on regular problems with curved boundary that converges as $O(h^p)$, where h is the element size and p the polynomial basis order. To get optimal

rate of convergence when curved interfaces are represented with level-set, the obvious solutions would be to increase the order of the level-set representation near the zero iso-values. In this case, the integration problem can become quite difficult : within an element, the iso-zero surface can have complex shape and topology.

The alternative approach that we propose consists in representing the level-set on a finer mesh (figures 1 and 2), while keeping the level-set representation linear by element (without adding any degree of freedom). We start with the same mesh for the field representation and the level set. Each element of the initial mesh for the level-set that is crossed by the iso-zero is split (conformity of the mesh is not required for the level-set representation), and value of the level-set are computed on the new nodes. The procedure is done recursively up to a user defined maximum depth. Each element of the analysis is then linked to an octree-like partition where the level-set is represented and on which the previously described integration procedure is applied for each sub-element. The approximation of the zero surface is then improved at each level of refinement. The algorithm was implemented and tested on simple problems with curved boundaries. Numerical experimentation were done up to order three polynomial approximation and we show that optimal convergence rate can be achieved (figure 3), with an optimal depth of recursion. Engineering rules to choose the optimal depth as a function of p are presented. Results on curved crack problems in two and three dimensions will be presented and compared to the results given in the literature [3], [4].

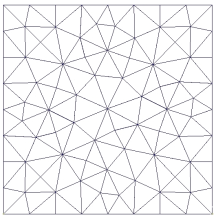

Figure 1: Analysis mesh

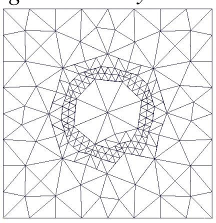

Figure 2: Integration mesh

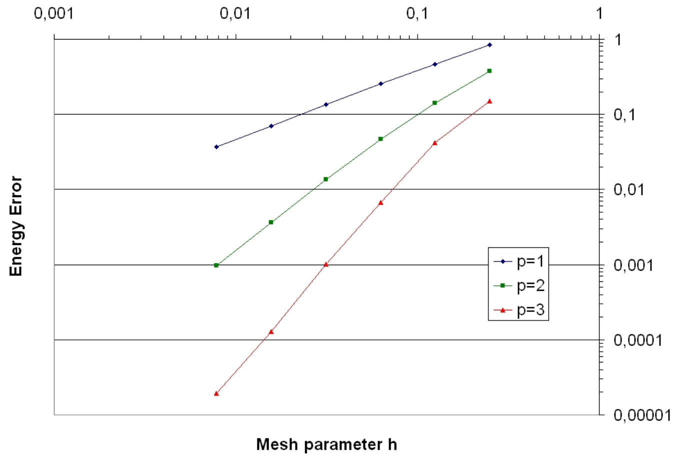

Figure 3: Energy error convergence

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