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ASPECTS OF THE MEAN-FIELD RELAXATION AND NUMERICAL SOLUTION OF THE FREE-DISCONTINUITY FORMULATION OF BRITTLE FRACTURE

*** Slav Dimitrov and Thomas Böhlke**

Chair of Continuum Mechanics, Institute of Engineering Mechanics,
University of Karlsruhe (TH), Kaiserstrasse 10, D-76128 Karlsruhe, Germany
dimitrov@kit.edu, http://www.itm.uni-karlsruhe.de

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ABSTRACT

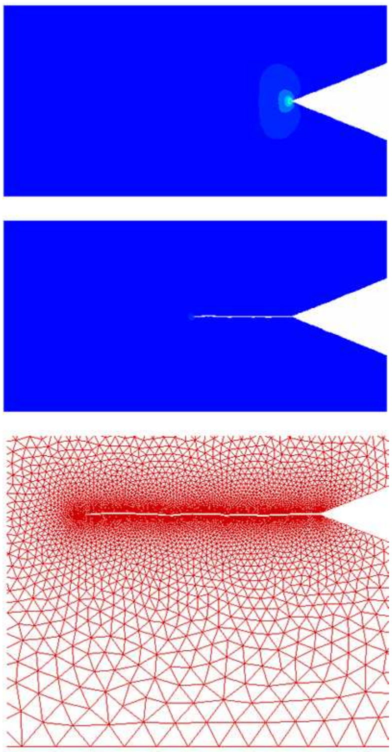

Figure 1. Calculated mode I crack propagation in a V-notched brittle specimen.

Sharing the conceptual line of [4,2] we regard the initiation and quasistatic growth of cracks in a brittle linearly elastic media as an energy minimization process, governed by a time-incremental analogue of the principle of least action. As comprehensively reviewed in [4,5] such a prospect on the formation and propagation of rectifiable surface-like strong discontinuities in solids allows to remedy two empirical deficits of the classical fracture mechanics [6]: (i) in the conditions for crack initiation - pre-existing crack is no longer required, *ab incunabulis*; and (ii) in the conditions for crack propagation - trajectory, branching, and propagation increments are *allowed by*, and *determined through*, the solution of the minimization problem. The existence issues addressed in the literature so far have undoubtedly shown that globally stable configurations of the solid - i.e. no bifurcations in the stress-strain history - admit vector-valued global solution of the variational crack propagation problem [3]. Motivated by these recent results we address the numerical solution of FRANCFORT-MARIGO formulation [4], using the finite element method. As already known [3], the function space for the FRANCFORT-MARIGO functional contains not only the associated displacement field but its discontinuity sets also, and hence, any attempt for numerical solution, based on a direct discretisation of this functional is attained by significant technical difficulties. Our approach instead (cf. [2]), is based on relaxation of the variational minimization problem by means of the Γ-convergent, elliptic, bi-convex, formulation proposed in [1]. If interpreted in a mean-field theoretical context the relaxed formulation represents the process of crack initiation/propagation as a second order, surface type of

phase transition, which in addition to the displacement field is governed by a phase-state field (phase field) serving the purpose of a continuous 'indicator' for the crack/no-crack (or, '0'-'1') state at every material point in the solid. Hence, the FRANCFORT-MARIGO problem of quasistatic crack propagation, reformulated this way, transforms into a problem for determination of the set of material points for which the phase field vanishes (the 0-th level set). In this talk we address the resolution of three important issues arising in the context of numerical calculation of the 0-th level set and the subsequent reconstruction of the crack surface. *First*, how by recasting the AMBROSIO-TORTORELLI's functional into a discrete, penalized, minimum-maximum problem one can avoid the undesirable scale effects expressed in terms of the characteristic size and domain-shape dependence of the calculated minimum. *Second*, how based on local averaging, one can construct an adaptive, re-meshing procedure in combination with a domain-shape update procedure for tracking the propagating 0-th level set. And *third*, how by a quadratic, 2-level approximation strategy one can reconstruct the crack surface from the calculated 0-th level set. We finally illustrate our approach by several 2-dimensional examples (cf. Fig. 1) for crack propagation in an initially homogeneous and isotropic, linearly elastic solid under plane strain condition.

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