8th. World Congress on Computational Mechanics (WCCM8)
5th European Congress on Computational Methods in Applied Sciences and Engineeering (ECCOMAS 2008)
June 30 –July 5, 2008
Venice, Italy

AN EMBEDDED/IMMERSED BOUNDARY METHOD FOR COMPRESSIBLE NAVIER-STOKES/LES EQUATIONS

*** Marco Kupiainen[1] and Pierre Sagaut[2]**

[1] Université Pierre et Marie Curie - Paris 6
IJLRA/UPMC, Boîte 162, 4 place Jussieu, F-75252 Paris cedex 5, France
kupiainen@lmm.jussieu.fr

[2] Université Pierre et Marie Curie - Paris 6
IJLRA/UPMC, Boîte 162, 4 place Jussieu, F-75252 Paris cedex 5, France
sagaut@lmm.jussieu.fr

Key Words: *Embedded boundary method, Immersed boundary method, Cartesian Grid method, Compressible Navier-Stokes, LES, Wall-function*

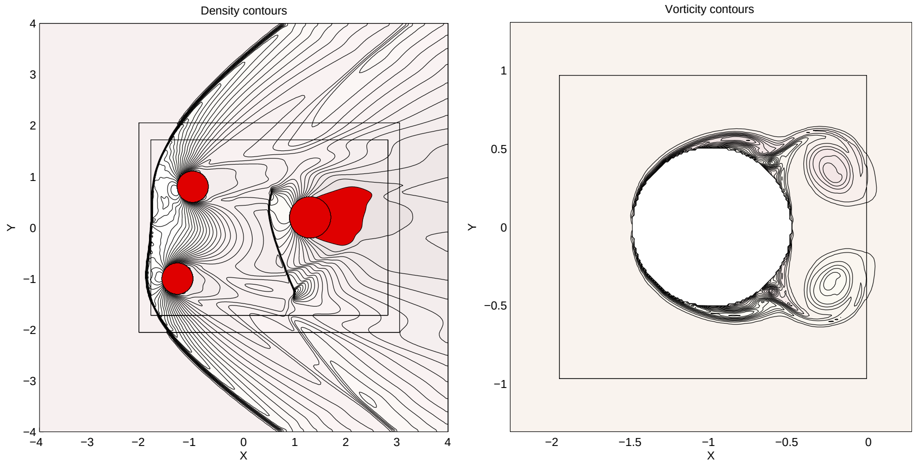
ABSTRACT

We present an Embedded Boundary Cartesian Grid method to set Dirichlet, Neumann and extrapolation boundary conditions for the compressible Navier-Stokes equations in two- and three dimensions. Of great interest is the coupling LES/wall-model/embedded boundary for simulating compressible flow applications. The main challenge with the embedded boundary method is to accurately satisfy the boundary conditions while retaining stability of the resulting scheme. The method is second order accurate using the second order extension of the Godunov/MUSCL scheme, even at the boundary. The explicit Runge-Kutta time-stepping scheme is stable with a time step determined by the grid size away from the boundary, i.e. the method does not suffer from small-cell stiffness. The benefits of a stretched grid in boundary layers are well-known. We use local mesh refinement near the boundary to compensate for the lack of stretching in the uniform grid. To mitigate the effects of unresolved flows using the embedded boundary method in boundary layers, we have used LES and adopted a meshless treatment of the walls by setting so called wall-conditions [6]. In comparison to 'classical' methods the boundary conditions are not imposed exactly when using the embedded boundary method, but to $\mathcal{O}(h^p)\ \ (p \geq 2)$.

Although not derived in the same manner, the proposed method is related to the Simultaneous Approximation Term (SAT) method [1]. The method we propose has previously been used to set boundary conditions for the second order wave equation [2,3,4] also a similar boundary method is used for the Euler equations [5].

Since the actual grid is not needed for this method, we believe that it is useful as a building block in a flow-simulator that deals with moving and/or deforming objects, complex objects and cases where generating the computational grid is tedious. An example of this could be a wing with morphing flaps.

We show examples of computations of flow around various objects in 2D and 3D. The results are compared with standard methods. Furthermore, we discuss the handling of thin objects i.e. when the embedded object is thinner than 2 cells and corners i.e. when ghost-points have multiple values.



(a) Mach 3 flow, three discs, density contours. $Re = 10^3$ using adiabatic-no-slip boundary condition.

(b) Vorticity contours Mach 0.5 $Re = 2.1 \cdot 10^6$ using wall-model to prescribe adiabatic friction velocity wall condition. The figure displays early time of an impulsive start problem.

Figure 1: Examples of results with the embedded boundary method using different wall-boundary conditions.

We show that for resolved flows the results are excellent. The method is second order accurate and stable. When the flow and/or geometry are very unresolved the results become unphysically unsymmetric. The result remind of those when the surface is rough. This roughness effect can to some extenent be mitigated by the use of LES and wall-model.

REFERENCES

[1] M. H. Carpenter, D. Gottlieb and S. Abarbarnel. “Time-Stable Boundary Conditions for Finite-Difference Schemes Solving Hyperbolic Systems: Methodology and Application to High-Order Compact Schemes”. *J. Comput. Phys.*, Vol. **111**, 220–236, 1994.

[2] H-O Kreiss, N. A. Peterson and J. Yström. “Difference approximations for the second order wave equation”, *SIAM J. of Num. Analys.*, Vol. **40**, 1940–1967, 2002.

[3] H-O Kreiss, N. A. Peterson and J. Yström. “Difference approximations of the Neumann problem for the second order wave equation”, *SIAM J. of Numer. Anal.*, Vol. **42**, 1292, 2004.

[4] H-O Kreiss, N. A. Peterson and J. Yström. “A second order accurate embedded boundary method for the wave equation with Dirichlet data”, *SIAM J. Sci. Comput.*, Vol. **27**, 1141, 2006.

[5] B. Sjögreen and N. A. Petersson. “A Cartesian Embedded Boundary Method for Hyperboloc Conservation Laws”. *Comm. Comput. Phys.*, Vol. **2**, 1199–1219, 2007.

[6] D. Monfort, S. Benhamadouche and P. Sagaut. *Meshless approach for wall treatment in Large-Eddy Simulation*, submitted to Physics of Fluids, 2007.