8th. World Congress on Computational Mechanics (WCCM8)
5th European Congress on Computational Methods in Applied Sciences and Engineeering (ECCOMAS 2008)
June 30 –July 5, 2008
Venice, Italy

Second Order Maximum-Entropy Approximation Schemes

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Key Words: *Maximum Entropy, Approximation Schemes, Boundary Conditions, Meshfree Methods*

ABSTRACT

A widely discussed problem in computational engineering is how to combine the advantages of both finite element method and meshfree methods. Whereas a clear advantage of meshfree methods is for example the possibility of refinement by arbitrary placement of new nodes regardless any mesh most of these methods suffer from drawbacks especially close to the boundary, where special strategies such as point collocation method or Nitsche's method are required in order to impose boundary conditions. For a long time benefitting from the characteristic advantages of both meshfree and meshbased methods was possible only by means of intricate blending methods.

In [1] a novel approximation scheme using so called first order maximum-entropy (max-ent) shape functions was developed. A key feature of this approach is the capacity of uniting the advantages of both meshfree and meshbased methods by enabling a weak Kronecker-Delta-property in the absence of any mesh. By appropriate parameter choice even a seamless transition between linear finite elements and meshfree approximation is possible and in addition to that max-ent shape functions satisfy a positivity-requirement, which is a key feature in dealing with dynamical problems as discussed in [2].

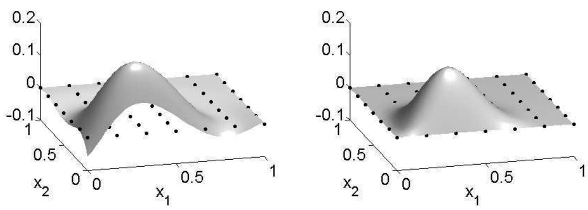

Figure 1: Second order moving least square (left) and max-ent (right) interior shape functions: Clearly only the max-ent shape function is positive throughout the whole domain and allows to impose boundary conditions straightforward

However, whereas common meshfree methods can be extended to higher order consistency only at the sacrifice of positivity of the shape functions, which is an obvious drawback for dynamical problems, max-ent approach allows an extension to second order consistency still preserving positivity and a

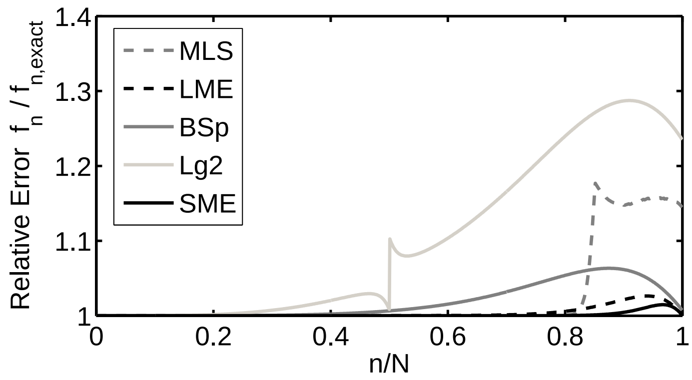

Figure 2: Relative error of the eigenfrequency spectrum of an elastic one dimensional rod applying different approximation schemes: Quadratic Finite Elements (Lg2), Moving Least Square Method (MLS) and B-spline Method (Bsp) as well as second order Max-Ent Method (SME) and linear Max-Ent Method (LME)

weak Kronecker-Delta-property (cf. Fig. 1). As a consequence imposing boundary conditions on second order max-ent shape functions is straightforward and the approximation power in dynamical problems is superior as demonstrated in Fig. 2 using the example of the eigenfrequency analysis of an elastic rod. Furthermore second order max-ent approximation represents the only non-negative higher order approximation scheme for unstructured data input in $\mathbb{R}^n$.

Doubtlessly a crucial point in the assessment of an approximation scheme is computational cost. In several examples it was found that numerical integration of max-ent shape functions can be done by means of clearly less integration points than numerical integration of common meshfree shape functions. In addition to that evaluation cost at each integration point is moderate so that max-ent approach unites remarkable approximation power with competitive computational cost.

The presentation will comprise an introduction into max-ent approach in general and especially into second order max-ent approximation. Advantages of the method will be oulined by means of several examples covering statical and dynamical problems of solid mechanics as well as fluid type model problems.

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