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ARTIFICIAL NEURAL NETWORKS FOR THE SOLUTION OF OPTIMAL SHAPE DESIGN PROBLEMS

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ABSTRACT

Optimal shape design is considered a field of great interest for industrial applications. The goal in this class of problems is to optimize a performance criterion dependent on the geometry of some body, while satisfying the possible constraints. For instance, a typical optimal shape design problem in aeronautics is to determine an airfoil section with given lift and reduced drag.

Mathematically, optimal shape design problems belong to the more general class of variational problems [1]. The aim of a variational problem is to find a function which is the optimal (minimal or maximal) value of a specified objective functional. Optimal shape design problems are usually defined by integrals, ordinary differential equations or partial differential equations.

While some simple optimal shape design problems have analytical solution, general problems can only be solved numerically by using direct methods [1]. The fundamental solution approach of direct methods comprises three independent steps: (i) selection of a parameterized function space representing the possible solutions to the problem; (ii) formulation of the variational problem, by choosing a suitable objective functional; (iii) solution of the reduced function optimization problem with deterministic or stochastic methods.

Regarding the first step, the function space chosen to represent the shape of a body is traditionally that spanned by a Bézier polynomial [2]. However, this approach presents several drawbacks. Certainly, Bézier curves do not have adequate approximation properties, and they are numerically instable for large numbers of control points. This also might cause the presence of many local optima in the objective function, even for very simple problems.

In the last decades, research in artificial intelligence has found new models of knowledge representation and processing that are closer to 'human-like' reasoning. Neural networks is a powerful paradigm there, which has been successfully applied to many applications in engineering. A neural network is a biologically inspired computational model consisting of a network architecture composed of artificial

neurons [3]. This structure contains a set of free parameters, that are adjusted to perform certain tasks. The multilayer perceptron is an important model of neural network, defined as a feed-forward network architecture of perceptron neuron models [3].

Within a variational formulation for the multilayer perceptron, the learning problem for that neural network consists in finding a function which is an extremal for some functional [4]. Moreover, a variational formulation for the multilayer perceptron provides a direct method for the solution of general variational problems, in any dimension and up to any degree of accuracy. Typical examples include optimal control, inverse problems and optimal shape design.

In this work we investigate the feasibility of neural networks for solving optimal shape design problems. Here the function space chosen to represent the shape of a body is that spanned by a multilayer perceptron. Indeed, a multilayer perceptron with only one hidden layer of sigmoid neurons and an output layer of linear neurons provides a general framework for approximating any function from one finite dimensional space to another up to any desired degree of accuracy, provided enough hidden neurons are available. In this sense, multilayer perceptron networks are a class of universal approximators [5].

In order to validate this numerical method we train a multilayer perceptron to solve two optimal shape design problems in the aeronautical industry with analytical solution. The neural network results are then compared against the exact values. In particular we determine the optimal shape of a body of revolution for minimum drag in a Newtonian flow [6], and the optimal shape of a body of revolution with a given volume and for minimum drag in a Newtonian flow [6]. The former is an unconstrained problem, while the later is a constrained one.

The neural networks results show a very good agreement with the analytical solutions. More specifically, the deviation between the numerical and the exact values is less than 0.1% for the two problems considered. Also, the objective function in both cases does not show to contain local optima, since the quasi-Newton training algorithm always produces convergence to the global optimum.

Future work will be concentrated on the solution of more complex optimal shape design problems by means of neural networks. Special interest is devoted to the optimal design of a wing section for given lift and reduced wave drag in an inviscid compressible flow.

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