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A MUSTA strategy to discretize the Coulomb friction term in avalanches models.

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ABSTRACT

In this work, we study the discretization by finite volume methods of Savage-Hutter type models for avalanches and submarine avalanches (see [2], [4]).

Savage and Hutter propose in [4] a Saint Venant type model, in local coordinates over a plane with constant slope, to study avalanches. In [2] a generalization to study submarine avalanches , in local coordinates over a general bottom, is presented.

One of the mean characteristics of these models are the presence of a Coulomb friction term. The Coulomb friction term impose that the velocity of the moving rock layer is zero if the sum of all forces have a contribution minor than a critical value. Otherwise, the definition of Coulomb term depends on the angle of repose of the material. The presence of this term, impose from the numerical point of view two difficulties: the first one is to preserves the stationary solutions of the model (well-balance property) and the second one is how to impose numerically the two possible definitions of the Coulomb term.

In [2] a kinetic scheme is presented to discretize the Savage-Hutter model for avalanches. In this paper Mangeney et al. propose a semi-implicit discretization of the Coulomb term that allows to impose numerically both definitions of the Coulomb term. This discretization does not provide, in general, a well-balance scheme.

In this work, we follow the technique proposed in [3] mixed with a MUSTA strategy for non-conservative hyperbolic systems proposed in [1], in order to ensure the well-balance property of the numerical scheme, as well as, the imposition of both definitions of the Coulomb term. Both properties are relevant when simulating submarine avalanches in order to be coherent with the movement of both layers: water and rock layer.

We propose a finite volume method that exactly preserves water at rest with no movement of the rock layer when the angle of its surface is smaller than the angle of repose. And all stationary solutions are

preserved up to second order. Finally we also propose an extention of high order by state reconstructions.

Finally, we present several tests where we study aerial and subaerial avalanches.

REFERENCES

[1] M.J. Castro, C. Parés, A. Pardo and E. F. Toro , *Well-balanced high-order MUSTA Schemes for non-conservative hyperbolic systems.* Proceedings of ENUMATH 2007 (to be published).

[2] E.D. Fernández-Nieto, F. Bouchut, D. Brech, M.J. Castro, A. Mangeney, *A new Savage-Hutter type model for submarine avalanches and generated tsunami*. Submitted (2007).

[3] A. Mangeney-Castelnau, J.P. Vilotte, M.O. Bristeau, B. Perthame, F. Bouchut, C. Siomeoni, S. Yerneni, *Numerical modeling of avalanches based on Saint Venant equations using a kinetic scheme*. J. Geoph. Res. vo. 108, n. B11, 2527 (2003).

[4] S.B. Savage, K. Hutter, *The dynamics of avalanches of granular materials frominitiation to run-out*, Acta Mech. 86, 201-223 (1991).