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THEORY AND SIMULATION OF A BRITTLE DAMAGE MODEL IN THERMOELASTODYNAMICS

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ABSTRACT

The response of brittle materials subjected to extreme thermomechanical loading is investigated. In particular, the derivation of an initial boundary value problem describing the behavior of bulk carbon subjected to temperature gradients and pressures commonly found in the operating environments of solid rocket motors is presented. Experimental evidence suggests that these operating conditions are so severe that erosion, and ultimately component failure, occurs on a regular basis . Therefore, we are motivated to include the effects of changing microstructure in this model as well; however, a method which explicitly accounts for loss of mass or fragmentation is left for future studies.

The natural setting for capturing the physics of this problem is thermoelasticity. Viscous forces and plastic deformation are ignored due to the nature of the loading we wish to explore, under which their effects are expected to be negligible. The change in microstructure considered in this model is limited to the evolution of isotropic microcracking, which is described by a scalar phenomenological damage variable. Microcracking is assumed to be irreversible, and as such no healing is considered. This simple model is viewed as a starting point for the study of the complex processes involved in the degradation of bulk carbon under such strenuous conditions. Certainly, there exists a rich literature devoted to the topic of selecting appropriate damage models, and in the future these will be reviewed for incorporation into the present work. At this point, the dynamic relation between the elastic, thermal, and microstructural states is enough to warrant its own investigation. This is especially true for the numerical simulations.

Damage evolution is extended from a simple brittle fracture constitutive relation (see, e.g., Caiazzo and Costanzo, 2000).. Letting $\varphi(X,t)$ be the scalar damage variable representing microcrack density (see, for example Krajcinovic, 1996), the evolution equation is written as:

$$\dot{\varphi} = \eta \left\langle G - G_{cr} \right\rangle . \tag{1}$$

Here G is the free energy release rate with respect to the growth of microcracks, and $\langle \cdot \rangle$ is the usual positive-part operator. The additional material properties of η and G_{cr} are the crack growth viscosity coefficient and energy release rate threshold, respectively. These coefficients may be properties of the deformation gradient, temperature, and/or temperature. Such dependence is critical to this model's ability to describe the variation in behavior observed in brittle materials over at different temperatures. Exact forms for such constitutive properties are topics of active debate, if known at all; proposed dependencies for the specific case of bulk carbon are considered.

The final initial boundary value problem results in a system of eight fully-coupled non-linear PDE, where the displacements, velocities, temperature, and damage variable are the unknowns. The system is very stiff, due to the hyperbolic nature of the momentum balance and parabolic nature of the energy balance. Damage evolution leads to an ODE in the damage variable, and thus the usual problem of solution localization. This is addressed through the simple implementation of a minimum mesh diameter, based on the grain size of the domain, in the numerical simulations.

A hybrid finite difference/finite element method is implemented to solve the resulting initial boundary value problem in time and space. The fully implicit BDF2 method is used for the time integration to handle the stiff nature of the problem. A combination of globally continuous polynomials are used to approximate the macro fields of displacement and temperature, while element-wise continuous polynomials are used to approximate the damage variable. This combination of finite element techniques appears to satisfy a stability condition within the problem (which has yet to be rigorously identified), and supposes that globally discontinuous elements are a viable method for addressing localized damage variables. Adaptive mesh refinement is implemented to keep the problem size reasonable; a custom error estimator has been proposed and refinement schemes have been studied. The overall algorithm is implemented in a custom C++ program, which takes of advantage of parallel processing and relies extensively on the deal.ii finite element library.

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