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Enhancing accuracy and convergence order of high order methods with adaptive techniques

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ABSTRACT

This presentation aims at pointing out the duality between numerical schemes and computational meshes in the sense that utilizing appropriate meshes improve numerical schemes. Indeed, a main advantage of unstructured meshes, *i.e.,* tetrahedral meshes, is their flexibility for generating elements with large shape and size variations everywhere in the domain. Consequently, processes to automatically control the mesh generation are of main interest. This is the role of mesh adaptation.

Here, we present two mesh adaptation approaches and their impact on the mixed-element-volume method [1]. The first one is a multi-scale mesh adaptation strategy that impacts the accuracy and the global order of convergence of the solution. The second is a goal-oriented approach that impacts the accuracy and the convergence of scalar output such as drag. We will also illustrate other numerical benefits of adaptive meshes on solutions.

In the context of steady CFD computations, some numerical experiments point out that only a global mesh convergence order of one is numerically reached on a sequence of uniformly refined meshes although the considered numerical scheme is second order. This is due to the presence of genuine discontinuities or sharp gradients in the modelled flow. In order to address this issue, a continuous mesh adaptation framework is proposed based on the metric notion [2]. It relies on a $\mathbf{L}^p$ control of the interpolation error for twice differentiable functions. This theory provides an optimal bound of the interpolation error involving the Hessian of the solution. From this estimate, an optimal metric is exhibited to govern the adapted mesh generation. As regards steady flow computations with discontinuities, a global second order mesh convergence is predicted. To this end, a higher order smooth approximation of the solution is reconstructed providing an accurate and reliable Hessian evaluation. Several numerical examples in two and three dimensions illustrate that the global convergence order is recovered using this mesh adaptation strategy.

Then, we propose a goal-oriented error analysis for a 3D steady Euler model for anisotropic mesh adaptation. The upwind mixed-element-volume approximation is considered for flow discretisation. An a priori analysis is applied in order to exhibit the influence of local interpolation error. The specification of the scalar output, that we aim at computing accurately, is performed by means of an adjoint state. An optimal metric including stretching is obtained by a calculus of variation. This analysis proposes a complete anisotropic formulation of the problem that differentiate it from previous work such as [3]. The efficiency of the approach is illustrated on an adaptive simulation to predict the flow around an aircraft.

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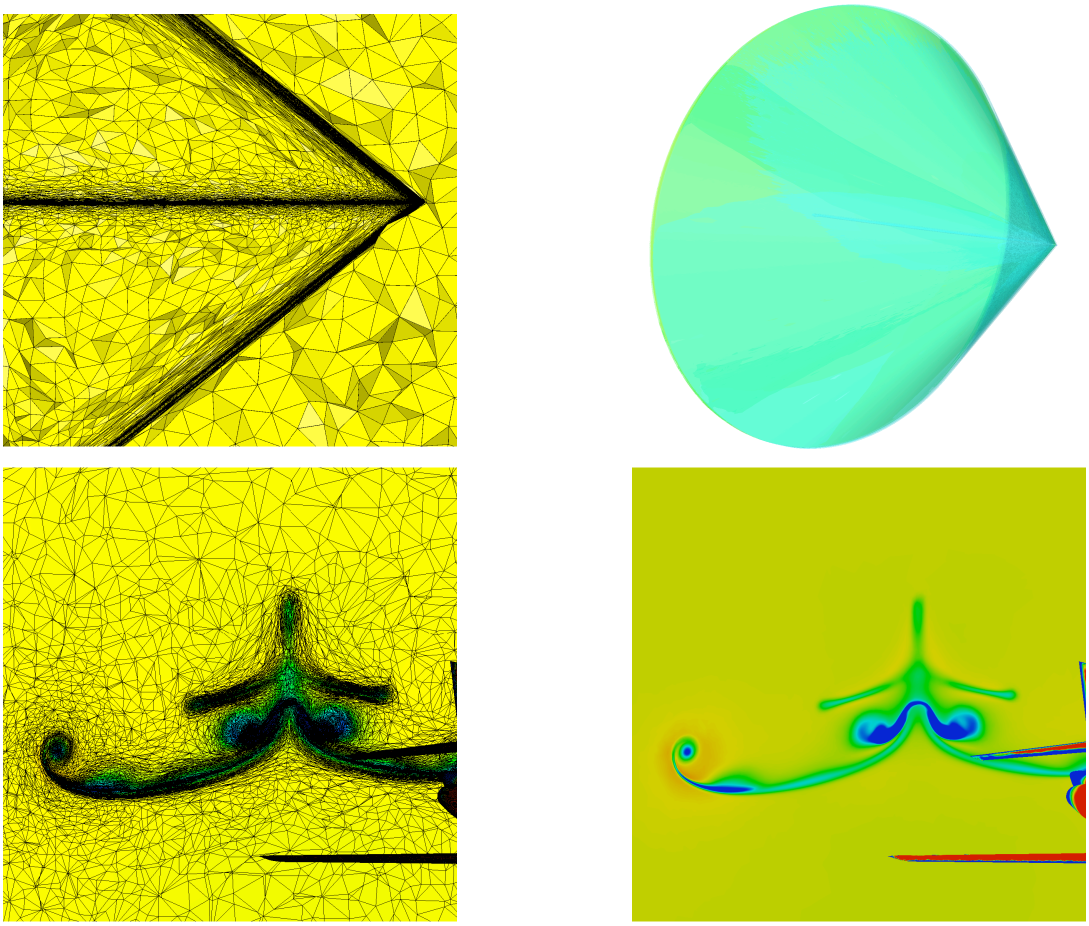

Figure 1: Aeronautic simulations with anisotropic adapted meshes. Top, study of the sonic boom emitted by a supersonic aircraft. Bottom, subsonic flow with shocks around a business jet (Mach number).