8th. World Congress on Computational Mechanics (WCCM8)
5th European Congress on Computational Methods in Applied Sciences and Engineeering (ECCOMAS 2008)
June 30 –July 5, 2008
Venice, Italy

A Dual Preconditioned Projected Conjugate Gradient algorithm for solving contact problems

***Nicolas Tardieu [1 2], Eric Chamberland [1], Fabien Youbissi [1]**

[1] GIREF
Université Laval,
Québec, Canada, G1K 7P4
{ntardieu, echamberland, fy-oubiss}@giref.ulaval.ca

[2] EDF R&D
1, avenue du Gl. de Gaulle
92141 Clamart Cedex
France
nicolas.tardieu@edf.fr

Key Words: *Contact, Conjugate Gradient, Preconditioner.*

ABSTRACT

The aim of this paper is to present a Dual Preconditioned Projected Conjugate Gradient algorithm for solving unilateral problems with special emphasis on the preconditioning phase.

The Projected Conjugate Gradient method comes from the field of convex optimization, where it is used to solve minimization problems with linear inequality constraints. Previous work has shown that it can be used efficiently for contact problems. In the present work, we develop an algorithm for continuum solid mechanics contact (similar approach is presented in [1] with application to granular media) based on the dual formulation of the contact problem (in the papers [2, 3], the primal formulation is considered) with a special preconditioner leading the mesh-size independent convergence.

We consider a contact problem on a part Γ_C of the boundary of an elastic body Ω. Once the problem is discretized by appropriate finite elements and dualized by introducing a Lagrange multiplier λ, it can be written as :

$$\min_{\lambda \geq 0} \mathcal{H}(\lambda) \tag{1}$$

where :

- $\mathcal{H}(\lambda) = \frac{1}{2}\lambda^T \boldsymbol{B}\boldsymbol{K}^{-1}\boldsymbol{B}^T\lambda - \lambda^T \boldsymbol{B}\boldsymbol{K}^{-1} + \frac{1}{2}F^T\boldsymbol{K}^{-1}F$
- $\boldsymbol{K}$ is the stiffness matrix
- $\boldsymbol{B}$ is the discretized gap operator on Γ_C
- F is the right-hand side loading vector.

The problem (1) can be rewritten by eliminating the positivity constraint on λ by adding the indicator function $\mathcal{I}_{\Delta^+}$ of the convex set $\Delta^+ = \{\lambda \geq 0\}$ to the functional $\mathcal{H}$.

$$\min_{\lambda} \left(\mathcal{H}(\lambda) + \mathcal{I}_{\Delta^+}(\lambda)\right) \tag{2}$$

The Conjugate Gradient algorithm can then be applied. Care has to be taken to the non-differentiable character of functional to be minimized, that leads to the use of subgradients.

As usual with iterative methods, it is highly necessary to use a preconditioner in order to speed up convergence [4]. We propose here to use a lifting technique from $H^{1/2}(\Gamma_C)$ into $H^{-1/2}(\Gamma_C)$ by solving the following auxiliary problem :

$$\begin{cases} \boldsymbol{K}\boldsymbol{u} + \boldsymbol{B}_{act}^T \tilde{d} = \boldsymbol{0} \\ \boldsymbol{B}_{act}\boldsymbol{u} = d \end{cases} \tag{3}$$

where :

- $\boldsymbol{B}_{act}$ is the discretized gap operator restrained to active constraints.
- $\tilde{d}$ is the preconditioned search direction d of the Conjugate Gradient.

This preconditioner is of Dirichlet kind, with strong relation with the Domain Decomposition FETI preconditioner [6].

We show that this preconditioner highly speeds up the resolution and brings a nice mesh-size independent convergence. Finally, a genuine but efficient parallel implementation of this method is presented.

References

[1] M.RENOUF, P.ALART - *Conjugate gradient type algorithms for frictional multi-contact problems: applications to granular materials*, Comput. Methods Appl. Mech. Engrg. **194**, 2005

[2] H.-O.MAY - *The conjugate gradient method for unilateral problems*, Comput. Struct. **22**, 1986

[3] G.DILINTAS, P.LAURENT-GENGOUX, D.TRYSTRAM - *A conjugate projected gradient method with preconditioning for unilateral contact problems*, Comput. Struct. **29**, 1988

[4] Y.SAAD - *Iterative Methods for Sparse Linear Systems*, SIAM, 2005

[5] M.S.BAZARAA, H.D.SHERALI, M.SHETTY - *Nonlinear Programming - Theory and Algorithms*, John Wiley and Sons, 1993

[6] C.FARHAT, F.-X.ROUX - *A method of finite element tearing and interconnecting and its parallel solution algorithm*, Int. J. Numer. Meth. Engng. **32**, 1991