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# An Introduction to the Level Set Methods and its Applications

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**ABSTRACT**

Finding a mathematical model which describes the evolution of an interface (in this context, an interface is understood as the boundary between two separate and closed regions, each one having a volume measure different from zero) over the time, like a burning flame or breaking waves, can be a challenging problem. The main difficulties arise when sharp corners appear or different parts of the interface are split or merged, [1]. That kind of interface can be modeled as the embedded zero level set of an implicit time-dependent function. So, the evolving interface can be followed by tracking the zero level set of that implicit function.

The above briefly described technique, known as the Level Set Method was introduced by Osher and Sethian, [2]. The idea behind this method [3] is to start with a closed curve, $\Gamma$, in two dimensions (or a surface in three dimensions) and allow the curve to move perpendicularly to itself from an initial speed, $F$. If the sign speed is preserved, the location of the propagating front is computed as the arrival time $T(x, y)$ of the front as it crosses the point $(x, y)$. In this case, the equation that describes this arrival time is given as:

$$|\nabla T|\, F = 1, \qquad T = 0 \quad on \quad \Gamma.$$

In the general case, the interface can not be considered as the level set of a spacial-dependent function because the arrival time is not a single-valued function. The way to address this difficulty is to represent the initial curve implicitly as a zero level set of a function in one higher dimension. So, at any time, the front is given by the zero level set of the time-dependent function, $\Phi$, referred to as the level set function. Mathematically, the set written as:

$$\{x(t) : \Phi(x(t), t) = 0\}$$

represents the interface at time $t$. Applying the chain rule and some algebraic manipulation, we can obtain the level set equation:

$$\Phi_t + |\nabla\Phi| F = 0, \qquad \Phi(x(0), 0) = \Gamma.$$

This method is a powerful mathematical and computational tool for tracking the evolution of curves/surfaces along image sequences. The main advantage came from a different approach similar to the Eulerian formulation. Instead of tracking a curve through time, the Level Set Method evolves a curve by updating the level set function at fixed coordinates through time, [4]. This approach [3], which handles topological merging and breaking in a natural way, is easily generalized to any other dimensional space and do not require that the moving front behaves as an explicit function.

The Level Set Method has been widely applied in different areas [3] like geometry, grid generation, image enhancement and noise removal in image processing, shape detection and recognition in image analysis, combustion and crystal growth analysis, among others.

Our purpose is to use this approach in the segmentation of structures represented in medical images. This task is very important for an adequate medical diagnosis, for example, in the location of anatomical structures or even in the analysis of its motion. The main difficulties [4] are due to the large variability in the structure shapes and the considerable quantity of noise that acquired images can have.

We designed a computational platform in C++, using Visual Studio .NET 2005 environment, and integrated in it the computational library OpenCV (http://sourceforge.net/projects/opencvlibrary) that gave us the possibility for using a great quantity of basic algorithms available for image processing and analysis. Now, we are implementing the above described technique to segment anatomical structures represented in medical images. Our final goal is to estimate the material properties of anatomical structures segmented and tracked along image sequences.

In this presentation, we are going to describe the Level Set methodology, exhibit some of its possible applications, present our segmentation method under development and show some of its experimental results.

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