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Localized Artificial Viscosity and Diffusivity Scheme for Capturing Discontinuities on Curvilinear and Anisotropic Meshes

*** Soshi Kawai[1] and Sanjiva K. Lele[2]**

[1] Center for Turbulence Research, Stanford University
488 Escondido Mall, Stanford, CA 94305, USA
skawai@stanford.edu

[2] Dept. of Mechanical Engineering and Aeronautics and Astronautics, Stanford University
Durand, Stanford, CA 94305, USA
lele@stanford.edu

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ABSTRACT

In the present study, simple and efficient localized high-wavenumber biased artificial viscosity and diffusivity in a multi-dimensional curvilinear coordinate framework are proposed for capturing shock waves and contact surfaces using a high-order compact scheme. The method is intended to use for the application of large-eddy simulation (LES) to compressible transitional and turbulent flows.

The Navier-Stokes equations including artificial viscous and diffusive terms[1,2] are solved in generalized curvilinear coordinates, where spatial derivatives are evaluated by the 6th-order compact difference scheme[3]. A 4th-order Runge-Kutta method is used for temporal integration. The 8th-order low-pass spatial filtering scheme is used to ensure numerical stability.

The original 1-D formulations of artificial viscosity[1] and diffusivity[2] are reformulated and generalized to construct a consistent multi-dimensional method on curvilinear and anisotropic meshes. The reformulation reduces the computational costs for more practical use while maintaining the properties of the original 1-D formulations. The generalized artificial shear and bulk viscosities μ_s and μ_b and artificial diffusivity in the mass equations χ_ρ on a curvilinear coordinate system are defined by:

$$\mu_s = C_\mu^s \alpha_r, \quad \mu_b = C_\mu^b \alpha_r, \quad \chi_\rho = C_\rho \beta_r, \tag{1}$$

$$\alpha_r = \rho \overline{\left| \sum_{l=1}^{3} \Delta_l^{r+2} \left[\sum_{m=1}^{3} \left(\frac{\partial \xi_l}{\partial x_m} \right)^2 \right]^{r/2} \frac{\partial^r S}{\partial \xi_l^r} \right|}, \quad \beta_r = \frac{a_0}{c_p} \overline{\left| \sum_{l=1}^{3} \Delta_l^{r+2} \left[\sum_{m=1}^{3} \left(\frac{\partial \xi_l}{\partial x_m} \right)^2 \right]^{r/2} \frac{\partial^r |\nabla s|}{\partial \xi_l^r} \right|}, \tag{2}$$

where C_μ^s, C_μ^s and C_ρ are user-specified constants, ρ, S, a_0, c_p and s are the density, symmetric strain rate tensor, reference speed of sound, specific heat at constant pressure and fluid entropy, respectively. ξ_l refers ξ, η and ζ and x_m refers x, y and z when l and m are 1, 2 and 3, respectively. Δ_l is the grid spacing in the physical space along with the grid line in the ξ_l direction and is defined by $\Delta_l^2 = \sum_{n=1}^{3} \left(\frac{x_{n,i+1} - x_{n,i-1}}{2} \right)^2$, where $x_{n,i}$ refers x_i, y_i and z_i when n is 1, 2 and 3 and i is a node index in the ξ_l direction. r=4 is adopted in this study. The fourth derivatives in Eq. (2) are evaluated by sixth (C6) and fourth-order (C4) tridiagonal schemes and a fourth-order explicit scheme (E4) to evaluate the performance of each scheme. Further details on the methods can be found in the reference[4].

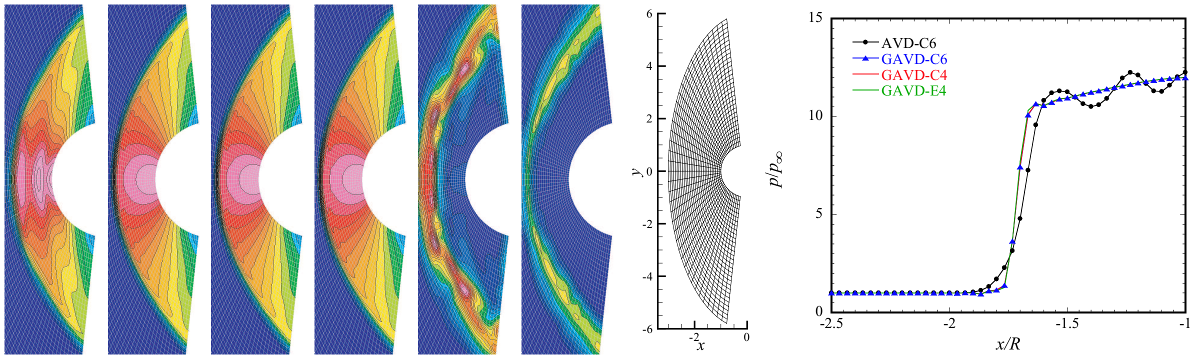

Figure 1: Numerical simulations of the Mach 3 blunt body flow. Pressure, 20 contours from 0.94 to 12, (a) AVD-C6, (b) GAVD-C6, (c)GAVD-C4, (d)GAVD-E4. Artificial viscosity, (e) AVD-C6, (f) GAVD-C6. (g) computational grid (every second grid point). (h) pressure profiles at centerline.

The proposed generalized methods are denoted GAVD-C6, GAVD-C4 and GAVD-E4 where the C6, C4 and E4 represent the schemes used to evaluate the fourth derivatives in the method. The original method is denoted AVD-C6 where the Δ^6 scaling is evaluated by $(\Delta x \Delta y \Delta z)^2$.

The method has been successfully applied to 1-D and 2-D shock related problems including shock-tube, Shu-Osher, double Mach reflection, oblique shock reflection and supersonic blunt body problems on isotropic and anisotropic Cartesian and curvilinear meshes. Here shows the results of the Mach 3 supersonic blunt body problem (Fig. 1). All the GAVD schemes show the converged solutions without spurious oscillations and almost identical results. The GAVDs capture the shock over 4-5 grid points. The shock location at x/R = -1.7 and post shock conditions are in good agreement with the reference solution. On the other hand, the AVD-C6 introduces relatively high artificial viscosity. This leads to smeared bow shock over 10 grid points and non-physical oscillation in the front bow shock.

The GAVD-E4 method has been applied to the practical problem of an under-expanded sonic jet injection into a supersonic crossflow and the results show the capability of the method for both capturing discontinuities (front bow shock, barrel shock, Mach disk and jet contact surfaces) and resolving turbulence within the framework of the LES.

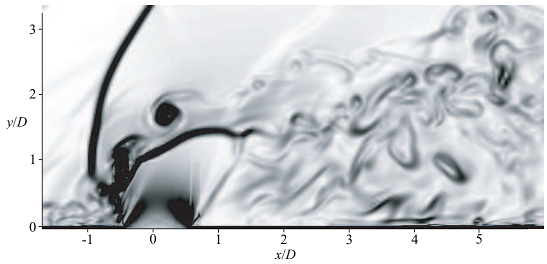

Figure 2: LES of a sonic jet in a supersonic crossflow: norm of density gradient.

Simple and efficient localized artificial viscosity and diffusivity schemes in a curvilinear coordinate framework have been successfully developed for the purposes of both capturing discontinuities and resolving turbulence by coupling with a high-order compact scheme. Since almost identical results are obtained by the GAVD schemes, the GAVD-E4 is an attractive choice to reduce the computational cost.

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