

## The LS-STAG Method : A new Immersed Boundary / Level-Set Method for the Computation of Incompressible Viscous Flows in Complex Geometries

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### ABSTRACT

Much attention has recently been devoted to the extension of Cartesian grid flow solvers to complex geometries by immersed boundary (IB) methods (see [4, 6] for recent reviews). In these methods, the irregular boundary is not aligned with the computational grid, and the treatment of the cells which are cut by the boundary remains an important issue. Indeed, the discretization in these cut-cells should be designed such that : (a) the overall accuracy of the method is not severely diminished and (b) the high computational efficiency of the structured solver is preserved.

Two major classes of IB methods can be distinguished on the basis of their treatment of cut-cells. Classical IB methods such as the momentum forcing method introduced in [3], use a finite volume/difference structured solver in Cartesian cells away from the irregular boundary, and discard the discretization of flow equations in the cut-cells. Instead, special interpolations are used for setting the value of the dependent variables in the latter cells. Thus, strict conservation of quantities such as mass, momentum or kinetic energy is not observed near the irregular boundary. Numerous revisions of these interpolations are still proposed for improving the accuracy and consistency of this class of IB methods [5]. A second class of IB methods (also called cut-cell methods, see *e.g.* [11]) aims for actually discretizing the flow equations in cut-cells. However, the calculation of numerical fluxes in cut-cells relies usually on techniques borrowed from methods that use curvilinear or unstructured body-conformal grids, and it is difficult to evaluate the negative impact of the cut-cells treatment on the computational cost of the simulations.

The purpose of this communication is to present a progress report on a new IB method for incompressible viscous flows which takes the best aspect of both classes of IB methods. This method, called the LS-STAG method, is based on the symmetry preserving finite-volume discretization on staggered Cartesian grids by Verstappen & Veldman [10], which has the ability to preserve on non-uniform Cartesian grids the conservation properties (for total mass, momentum and kinetic energy) of the original MAC method on a uniform staggered grid. The LS-STAG method has two distinctive features :

- In contrast to classical IB methods, flow variables are actually computed near the irregular boundary, and not interpolated. By representing the irregular boundary by its level-set function [8], we are able to efficiently calculate the numerical fluxes in the cut-cells.

- Our discretization has also the ability to discretize the fluxes in Cartesian and cut-cells in a unified fashion, ensuring that the 5-point Cartesian structure of the stencils is preserved. This property allows the use of an efficient black box multigrid solver for structured grids [9], yielding a highly efficient computational method.

Previous reports on the LS-STAG method can be found in [1, 2]. So far, the main difficulty we have encountered relies in the choice of the discretization in the cut-cells, where several possible formulations were tested, but none was found better than the others. In the present communication, the consistency and accuracy of the LS-STAG method has been dramatically improved by ensuring the strict conservation of global quantities such as total mass, momentum and kinetic energy in the whole fluid domain [7, 10]. To achieve these preservation properties up to the cut-cells, we had to precisely take into account the terms acting on the immersed boundary in the conservation equations, at both continuous and discrete levels. To our knowledge, these terms have always been neglected in past studies. As a result, the imposition of the kinetic energy conservation of the numerical scheme guided us towards the complete characterization of the pressure and convective fluxes in the cut-cells, and momentum conservation led to the unambiguous characterization of the diffusive fluxes.

The second-order of accuracy of the LS-STAG method is assessed on the Taylor-Couette problem. The ability to compute high Reynolds number viscous flow is illustrated on the vortex shedding past a circular cylinder. Comparisons with an unstructured solver in terms of CPU time and accuracy will be given. Finally, we will present some results for one of the most appealing features of IB methods : the ability to compute flows around moving objects on fixed cartesian grid, without the need for domain remeshing at each time step.

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