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VARIATIONALLY BASED PARTITIONED TRANSIENT AND QUASI-STATIC STRUCTURAL ANALYSIS PROCEDURES, PART I: ALGORITHM DESCRIPTION

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ABSTRACT

Variationally based solution algorithms for the partitioned analysis of transient and quasi-static structural mechanics problems are presented. A key property of the present algorithms is the judicious application of the d'Alembert-Lagrange principal equations. The paper includes three developments: (1) an implicit transient analysis algorithm that can be specialized to solve quasi-static problems while maintaining the same solution matrix profile; (2) a variational derivation of the FETI-DP-like multiprocessor solution method; and, (3) two complementary or dual solution algorithms by selecting which of the partitioned variables to be the primary solution vector. The algorithms presented herein subsumes many of previously developed partitioned solution procedures and offer new physical and/or numerical insight as each of the variational derivation process can be succinctly explained. Computational performance evaluations of the algorithms presented herein are presented in a companion paper, Part II.

THEORETICAL BASIS

The four-variable partitioned equations of motion for structures are derived in [1-2] as:

$$\begin{bmatrix} \mathbf{P}_\alpha(\mathbf{K}+\mathbf{M}\frac{d^2}{dt^2})\mathbf{P}_\alpha & \mathbf{0} & \mathbf{P}_\alpha\mathbf{B} & \mathbf{0} \\ \mathbf{0} & \mathbf{R}^T\mathbf{M}\mathbf{R}\,\frac{d^2}{dt^2} & \mathbf{R}^T\mathbf{B} & \mathbf{0} \\ \mathbf{B}^T\mathbf{P}_\alpha & \mathbf{B}^T\mathbf{R} & \mathbf{0} & -\mathbf{L}_f \\ \mathbf{0} & \mathbf{0} & -\mathbf{L}_f^T & \mathbf{0} \end{bmatrix} \left\{ \begin{array}{c} \mathbf{u} \\ \alpha \\ \lambda_\ell \\ \mathbf{u}_f \end{array} \right\} = \left\{ \begin{array}{c} \mathbf{P}_\alpha\,\mathbf{f} \\ \mathbf{R}^T\mathbf{f} \\ \mathbf{0} \\ \mathbf{0} \end{array} \right\} \qquad (1)$$

where $(\mathbf{M}, \mathbf{K}, \mathbf{R}, \mathbf{B}, \mathbf{L}_f)$ denote the substructural mass matrix, the substructural stiffness matrix, the rigid-body modes of partitioned floating substructures, the partition-boundary degrees of freedom Boolean matrix, and the boundary assembly matrix, respectively; $\mathbf{P}_\alpha = \mathbf{I} - \mathbf{R}(\mathbf{R}^T\mathbf{R})^{-1}\mathbf{R}^T$ is a projection matrix that filters out the rigid-body modes; and, $(\mathbf{u}, \alpha, \lambda_\ell, \mathbf{u}_f)$ represent the displacement pertaining to each partitioned substructure, the rigid-body displacement of each substructure, the localized interface Lagrange multipliers, and the global displacement along the assembled interface nodes, respectively.

The first row of the above equation represents the equations of motion for the deformable modes for each of the partitioned subsystems; the second is the D'Alembert principal equation for each of the partitioned subsystems [3]; the third is the interface kinematical constraints; and, the fourth is the Newton's third law along the interfaces in terms of localized Lagrange multipliers.

A key novel feature of the above partitioned four-variable equation is that one can employ different time integration algorithms for the partitioned displacement ($\mathbf{u}$) and the rigid-body displacement (α). Specifically, as the D'Alembert-Lagrange principal equation is characterized by zero frequency ($\omega = 0$), one may integrate α with an arbitrarily large stepsize without causing numerical stability. Hence, equation (1) can be effectively treated as

$$\begin{bmatrix} \mathbf{P}_\alpha(\mathbf{K}+\mathbf{M}\frac{d^2}{dt^2})\mathbf{P}_\alpha & \mathbf{0} & \mathbf{P}_\alpha\mathbf{B} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{R}^T\mathbf{B} & \mathbf{0} \\ \mathbf{B}^T\mathbf{P}_\alpha & \mathbf{B}^T\mathbf{R} & \mathbf{0} & -\mathbf{L}_f \\ \mathbf{0} & \mathbf{0} & -\mathbf{L}_f^T & \mathbf{0} \end{bmatrix} \left\{ \begin{array}{c} \mathbf{u} \\ \alpha \\ \lambda_\ell \\ \mathbf{u}_f \end{array} \right\} = \left\{ \begin{array}{c} \mathbf{P}_\alpha\,\mathbf{f} \\ \mathbf{R}^T\mathbf{f} - \mathbf{R}^T\mathbf{M}\mathbf{R}\,\ddot{\alpha}^p \\ \mathbf{0} \\ \mathbf{0} \end{array} \right\} \qquad (2)$$

where $\ddot{\alpha}$ is a predicted value.

Observe that if the partitioned displacement is integrated with an implicit time integration scheme for the case of dynamics, the resulting matrix profile remains the same as for the quasi-static case ($\mathbf{M}\frac{d^2}{dt^2} = \mathbf{0}$). This allows us to develop the same partitioned solution procedures for both the transient as well as quasi-static problems, which greatly simplifies the software development effort and, in particular, open new avenues for further algorithm improvements. We will present the details of the present partitioned solution procedures along with the algorithm performance treated in the companion paper, Part II [4].

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