# VARIATIONALLY BASED PARTITIONED TRANSIENT AND QUASI-STATIC STRUCTURAL ANALYSIS PROCEDURES, PART II: IMPLEMENTATION AND PERFORMANCE EVALUATION

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**Key Words:** Partitioned analysis, D'Alembert-Lagrange principal equations, variational solution algorithms

# ABSTRACT

An implicit partitioned transient analysis algorithm is presented for linear structural systems whose model consists of partitioned substructures. A key property of the present algorithm is the judicious application of the d'Alembert-Lagrange principal equations. The present implicit transient analysis algorithm can be reduced to solve quasi-static problems while maintaining the same solution matrix profile employed for transient analysis. The basic algorithm can be specialized to a family of solution algorithms by selecting which of the partitioned variables to be the primary solution vector. This generality is discussed in terms of the so-called primary and dual solution strategies as some of the variants are an intense subject in parallel computation community.

### THEORETICAL BASIS

The starting point of this work is the four-variable equation of motion presented in [1] for the solution of partitioned finite element structural systems in quasi-static and dynamic analyses. If the partitioned displacements are eliminated from that system and the substructure rigid body motions ( $\alpha$ ) are extrapolated, the following time discrete equation is obtained:

$$\begin{bmatrix} \mathbf{F}_{bb} & -\mathbf{B}^T \mathbf{R} & \mathbf{L}_f \\ -\mathbf{R}^T \mathbf{B} & 0 & 0 \\ \mathbf{L}_f^T & 0 & 0 \end{bmatrix} \begin{cases} \lambda_\ell \\ \alpha \\ \mathbf{u}_f \end{cases}^{n+1} = \begin{cases} \mathbf{b}_\lambda \\ \mathbf{b}_\alpha \\ 0 \end{cases}^n$$
(1)

where  $\mathbf{F}_{bb} = \mathbf{B}^T \mathbf{P}_{\alpha} \mathbf{K}_d^{\dagger} \mathbf{P}_{\alpha} \mathbf{B}$  is the dynamic flexibility matrix; *Rbold* is the substructural rigid-body modes;  $(\lambda, \alpha, \mathbf{u}_f)$  are the interface localized Lagrange multipliers, the rigid-body amplitudes and the interface displacements, respectively; and,  $\mathbf{b}_j$  are the time- discretized and/or predicted known vectors. Equation (1) can be solved using a FETI-like iterative algorithm[2] where the interface Lagrange multipliers  $(\lambda_\ell)$  appear as the principal unknowns, reducing the problem to the interface degrees of freedom.



# IMPLEMENTATION AND PARALLELIZATION

For the solution of general partitioned and distributed finite element problems using this approach [3-5] a modular and flexible object-oriented model has been defined including those entities appearing in the problem (substructures, interfaces, numerical components and algorithms) considering the sequence of operations that take place during a parallel solution process and paying special attention to the computational costs related with numerical operations and information transfer between the different processes. This software architecture has been implemented and tested for the structural case.

The partitioned problem is parallelized using distributed objects, which permit to create substructures on the different nodes of a cluster computer and manage a transparent access to their information like if they were resident in the master machine where the global interface problem is being solved. Finally, serial and parallel performance results on a PC-cluster machine are reported and analyzed for structural dynamics problems, studying the influence of different time-integration schemes and solution strategies on the complete solution process.

#### REFERENCES

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