8th. World Congress on Computational Mechanics (WCCM8)
5th European Congress on Computational Methods in Applied Sciences and Engineering (ECCOMAS 2008)
June 30 –July 5, 2008
Venice, Italy

IMPROVED APPROXIMATION FOR EXTERNAL ACOUSTIC-STRUCTURE INTERACTION VIA COMBINED RETARDED AND ADVANCED POTENTIAL, Part I: FORMULATION

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Key Words: *acoustic-structural interaction, Kirchhoff's retarded potential formula.*

ABSTRACT

New approximate models for external acoustics interacting with flexible structures are developed. The basic form of the present models is obtained by a combination of the Laplace-transformed retarded and advanced potentials. It is shown that the maximum attainable time-derivative of convergent approximate models is two, hence any attempt to include higher orders will lead to non-convergent models. The basic model is subsequently modified from the consistency considerations of the resulting frequency response functions. The resulting model is expressed in terms of a free parameter that represents the weight of the retarded vs. advanced potential characteristics, and consistent in terms of capturing the correct impulse response. Hence, the present approximate model is well suited for acoustic-field determinations and inverse acoustic identification applications.

PROPOSED APPROXIMATION

The starting point of the proposed acoustic-structure interaction is the following modified form of Kirchhoff's integral wave equation [1, 2]:

$$4\pi\epsilon\phi_{mod}(P,t) = -\int_S \{ \frac{1}{r}\frac{\partial\phi_{mod}(Q,\bar{t})}{\partial n} + \frac{1}{r^2}\frac{\partial r}{\partial n}\phi_{mod}(Q,\bar{t}) + \frac{1}{cr}\frac{\partial r}{\partial n}\dot{\phi}_{mod}(Q,\bar{t}) \} dS_Q$$

$$\begin{aligned} \text{Present modified potential :} \quad & \phi_{mod}(Q,\bar{t}) = \tfrac{1}{2}(1-\chi)\phi_r + \tfrac{1}{2}(1+\chi)\phi_a \\ \text{Retarded potential:} \quad & \phi_r = \phi(Q, t - r/c) \\ \text{Advanced potential:} \quad & \phi_a = \phi(Q, t + r/c) \end{aligned} \tag{1}$$

where ϕ_{mod} is the modified velocity potential, r is the distance from P to a typical point Q on the surface S; $\partial/\partial n$ denotes differentiation along the outward normal to S; ϵ is the solid angle that takes on $(1, 0.5, 0)$ depending on whether the point Q is within the acoustic domain, on the surface S, or inside the enclosed surface S, respectively; χ is the weighting parameter; and, $t_r = (t - \frac{R}{c})$ and $t_a = (t + \frac{R}{c})$ denote the retarded time and the advanced time, respectively.

A key idea of the present formulation is to attain a temporally stable, numerically accurate approximation of the following Laplace-transformed modified potential:

$$\bar{\phi}_{mod} = [\frac{1}{2}(1-\chi)e^{-sr/c} + \frac{1}{2}(1+\chi)e^{sr/c}]\,\bar{\phi}(Q,s) \tag{2}$$

After a careful expansion of of the above modified potential in Laplace-domain and substituting the various velocity potential expressions by the pressure $\mathbf{p}$ and the particle velocity on the structural surface $\mathbf{u}$, the following parameterized second-order external acoustic model has been formulated:

$$\begin{array}{c} s^2\mathbf{X}\mathbf{A}\overline{\mathbf{p}}(P,s) + sc(\mathbf{I}+\mathbf{X})\mathbf{A}_1\overline{\mathbf{p}}(P,s) + c^2\mathbf{B}_2\overline{\mathbf{p}}(P,s) = s^2\rho c\mathbf{X}\mathbf{A}\overline{\mathbf{u}}(P,s) + s\rho c^2\mathbf{A}_1\overline{\mathbf{u}}(P,s) \\ \mathbf{u} = -\partial\phi/\partial n, \qquad \mathbf{p} = \rho\dot{\phi} \\ \mathbf{B}_2\overline{\mathbf{p}}(P,s) = \int_S \frac{1}{r^2}\frac{\partial r}{\partial n}\overline{p}(Q,s)dS + 4\pi\varepsilon\delta(P-Q)\overline{p}(P,s) \\ \mathbf{A}\overline{\mathbf{u}}(P,s) = \int_S \overline{u}(Q,s)dS, \quad \mathbf{A}_1\overline{\mathbf{u}}(P,s) = \int_S \frac{1}{r}\overline{u}(Q,s)dS \end{array} \tag{3}$$

where $\mathbf{X}$ is the discrete counterpart of the weighting parameter, χ, of the retarded and advanced potentials. Details of selecting the weighting parameter will be discussed in the companion presentation, Part II [3].

CONCLUSION

An improved pressure-field governing equation has been developed for the modelling of external acoustic field interacting with flexible structures. The proposed model can be implemented using the available boundary integral matrices. The present parameterized model will be shown to yield improved accuracy than the DAA$_2$[4] approximation as discussed in Part II.

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