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AN ANISOTROPIC INELASTIC MODEL FOR PRECONDITIONING AND SOFTENING OF SOFT BIOLOGICAL TISSUES

*** Alexander E. Ehret, Mikhail Itskov**

Dept. of Continuum Mechanics,
RWTH Aachen University
52056 Aachen, Germany
ehret@km.rwth-aachen.de

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ABSTRACT

Mechanical testing of elastomers and biological tissues is often accompanied by softening phenomena like preconditioning in early load cycles or the well-known Mullins effect. The softening behavior has frequently been modeled in the framework of continuum damage mechanics for both rubber-like materials, see e.g. [1] and references therein, and soft biological tissues, e.g. [2]. In this paper, the material parameters of a recently presented hyperelastic model for fiber-reinforced materials [3] are allowed to evolve in order to reflect structural alterations inside the material. As a consequence, the model does not only predict softening and hysteresis but also accounts for residual deformations occurring after the material has been unloaded. The model is applied to simulate the preconditioning behavior of anisotropic soft biological tissues subjected to cyclic loading experiments. The results suggest that the general characteristics of preconditioning with different upper load limits are well captured.

Anisotropic inelastic model

We consider composite materials consisting of an isotropic matrix and n families of reinforcing fibers with orientation given by a unit vector $\boldsymbol{m}_i,\ i = 1, 2, ..., n$. The structural tensors

$$\mathbf{L}_0 = \frac{1}{3}\mathbf{I},\quad \mathbf{L}_i = \boldsymbol{m}_i \otimes \boldsymbol{m}_i,\quad i = 1, 2, ..., n \tag{1}$$

are introduced, where $\mathbf{I}$ denotes the identity tensor of second order. Considering the objectivity requirement, classical invariant theory and by applying the Cayley-Hamilton theorem, a strain-energy function for such a material can be represented in the following form [3]

$$W = \bar{W}\left(I_i, J_i, \mathrm{III}_\mathbf{C}\right),\quad I_i = \mathrm{tr}(\mathbf{C}\mathbf{L}_i),\quad J_i = \mathrm{tr}\left[(\mathrm{cof}\mathbf{C})\mathbf{L}_i\right],\quad \mathrm{III}_\mathbf{C} = \det\mathbf{C},\quad i = 0, 1, ..., n, \tag{2}$$

where $\mathbf{C}$ denotes the right Cauchy-Green tensor and $\mathrm{cof}\mathbf{C} = \mathbf{C}^{-\mathrm{T}}\det\mathbf{C}$. The arguments I_i, J_i and $\mathrm{III}_\mathbf{C}$ are convex with respect to the deformation gradient $\mathbf{F}$, its adjugate $\mathbf{F}^{-1}\det\mathbf{F}$ and its determinant $\det\mathbf{F}$ [4], respectively and form a reduced set of invariants. In a next step, so-called generalized invariants are introduced as linear combinations with non-negative weight factors $u_i^{(r)}$ and $v_i^{(r)}$, $i = 0, 1, ..., n$ (cf. [5]). Thus, an alternative representation of the strain-energy function can be given by

$$W = \tilde{W}\left(\tilde{I}_r, \tilde{J}_r, \mathrm{III}_{\mathbf{C}}\right), \quad \tilde{I}_r = \sum_{i=0}^{n} u_i^{(r)} I_i, \quad \tilde{J}_r = \sum_{i=0}^{n} v_i^{(r)} J_i, \; r = 1, 2, ... \tag{3}$$

The invariants I_i and J_i, $i = 1, 2, ..., n$, describe the change of the squared values of the length of a line element along $\boldsymbol{m}_i$ and the area of a surface element with normal $\boldsymbol{m}_i$, respectively. The influence of these changes on the strain energy is governed by the weight factors $u_i^{(r)}$ and $v_i^{(r)}$. In order to take into account inelastic phenomena, the weight factors $u_i^{(r)}$ and $v_i^{(r)}$ are considered as internal variables and are allowed to evolve independently. Accordingly, the elastic potential $(3)_1$ is extended to the free energy

Results

As a special case, the preconditioning behavior of a transversely isotropic soft biological tissue sample subject to uniaxial cyclic loading and unloading was considered. We chose an appropriate representation of the strain-energy function $(3)_1$ for soft biological tissues and set up evolution conditions and equations for the internal variables $u_i^{(r)}$ and $v_i^{(r)}$. Representative examples for both uniaxial loading in fiber and transverse direction and loading with increasing upper load limits are shown in Figure 1.

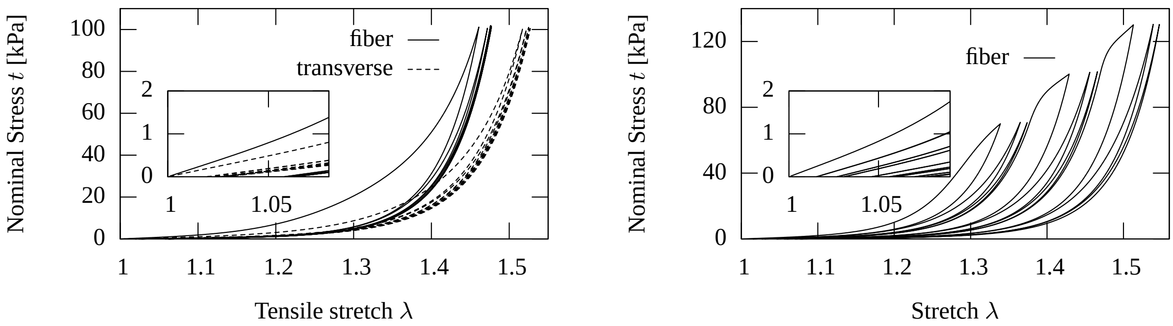

Figure 1: Preconditioning of an incompressible material sample in different directions (left) and with increasing load limits (right). Hysteresis, stabilization and permanent set are observed.

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