

## Dependence of Structure of Flow Simulation Results on Errors in Iteration Process

\* Itaru Hataue<sup>1</sup>

<sup>1</sup>Graduate School of Natural Science and Technology, Kanazawa University  
Kakuma-machi, Kanazawa 920-1192, Japan

Email: hataue@is.t.kanazawa-u.ac.jp, URL: <http://cpm.ec.t.kanazawa-u.ac.jp/>

**Key Words:** *Random Numerical error , Iterative Method , Incompressible Fluid.*

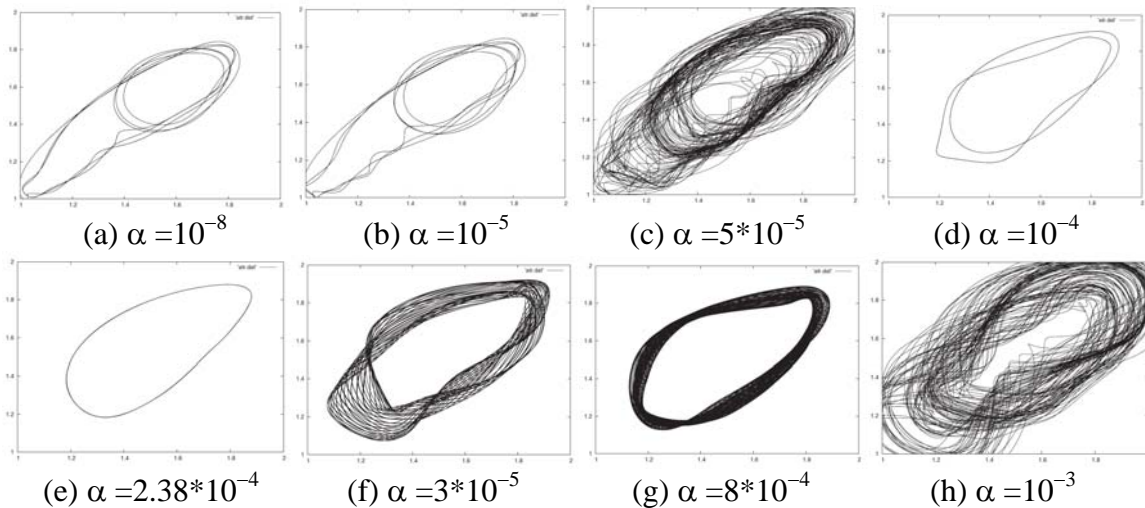
### ABSTRACT

In the present paper, we try to investigate the dependence of the structure of numerical solutions on insertion of random errors. The calculation model adopted in the present paper is a flow around a circular cylinder and the Reynolds number is 1500. The governing non-dimensional incompressible Navier-Stokes equations and the continuity equation are solved numerically by using the MAC(Marker-And-Cell) method[1] and the third-order upwind scheme.(The amplitude of introduced fourth order artificial viscosity term is denoted “ $\varepsilon$ ”.)

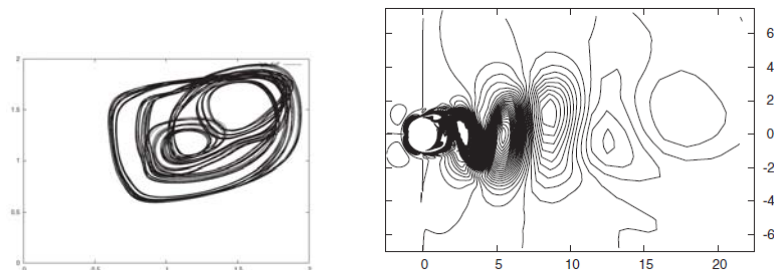
The Poisson equation(simultaneous equation) is solved using Gauss-Seidel scheme in which a convergence condition is determined by norm of error( $\alpha$ ). Random errors inserted by using the rough schemes in which large  $\alpha$  values are selected induce the appearance of stable spurious solutions[2]. We try to discuss the dependence of the structure of numerical solutions of incompressible fluid equations on insertion of random errors in solving simultaneous equations. Furthermore, the similarity and dissimilarity between the dependence of the structure of numerical solutions on the convergence condition( $\alpha$ ) and that on the forcibly added randomness given by using the pseudo-random number row are discussed. Here, we use the notation of “ $r$ ” for the amplitude of the forcibly added randomness.

Figure 1 shows the profiles of the trajectories reconstructed two-dimensionally from the time series of drag coefficient( $C_d$ ). The structure of periodic-6 in the cases of (a) and (b) breaks to the complicated one((c)) and simple periodic structure appear again in the cases of (d) and (e). When  $\alpha$  value becomes larger, tori appear((f) and (g)) and very complicated structure can be seen again(h) and calculation finally diverges where  $\alpha$  is a little larger than that of (h). This sequence of change of structure of attractors is similar to the bifurcation process caused by the structural instability in the deterministic dynamical system. On the other hand, vorticity field shown in Fig.2 is not so strange in spite of rough calculation of the Poisson equation of pressure. This result shows that the global structure of flow field is not so influenced by the convergence condition of the Poisson equation of pressure.

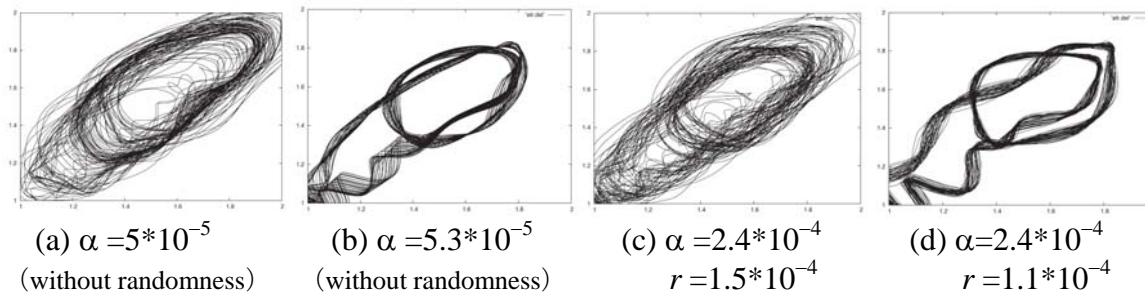
Figure 3 shows the comparison of profiles of the reconstructed trajectories with and without forcibly added randomness. It is clear that Fig.3-(a) is similar to Fig.3-(c) and Fig.3-(b) is similar to Fig.3-(d). This result shows that effects of errors inserted in the iteration processes in solving the Poisson equation to the structure of numerical solutions are similar to effects of forcibly added random ones.



**Fig.1 Profile of the reconstructed trajectories from the time series of drag coefficient in the case of  $\varepsilon = 0.17$ .**



**Fig.2 Profile of the trajectory and vorticity field in the case of  $\alpha = 5 \times 10^{-3}$ ,  $\varepsilon = 4.3$ .**



**Fig.3 Profile of the reconstructed trajectories showing the similarity of effects between  $\alpha$  and  $r$ .**

## REFERENCES

- [1] F. H. Harlow and J. E. Welch. "Numerical calculation of time-dependent viscous incompressible flow of fluid with free surface", Phys. Fluids, Vol. **8**, pp. 2182-2189, (1965).
- [2] I. Hataue "On the stabilizing effect of the condition for convergence in iteration processes in flow simulations", Proc. ECCOMAS 2004 CD-ROM (2004).