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STRESS CONSTRAINTS AGGREGATION IN STRUCTURAL TOPOLOGY OPTIMIZATION

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ABSTRACT

Topology optimization of structures problems have been usually stated in terms of maximum stiffness formulations due to the computing advantages they offer. However, different approaches that minimizes the weight under stress constraints are being analyzed since a few years ago to avoid some theoretical and numerical drawbacks of maximum stiffness formulations. The most usual way of imposing stress constraints is to check the stress value on the central point of each node of the mesh (the local approach) [1], [2]. Due to the high computing resources required by the local approach other different formulations have been developed that aggregate the effect of the local constraints in only one function (global approaches) [2].

In this abstract we propose a different technique that defines groups of elements and imposes a stress constraint over each one of them. This block aggregation procedure defines a fixed number of groups of elements. Once the number of groups is fixed the number of elements of the mesh is distributed equally into these blocks. Thus, all the blocks contain a similar number of elements.

The major idea of this approach is to apply over each block of elements (B_b) a global function like that proposed in [2]. Thus, the global constraint of the block b is defined as:

$$G_{b,KS}(\boldsymbol{\rho}) \;=\; \frac{1}{\mu}\, ln\left[\sum_{e\in B_b} exp^{\,(\mu\,(\widehat{\sigma}_e^{*}-1))}\right] - \frac{1}{\mu}\, ln(N_e^b) \;\leq\; 0 \quad \text{where} \quad \widehat{\sigma}_e^{*} = \frac{\widehat{\sigma}(\boldsymbol{\sigma}_e^h(\boldsymbol{\rho}))}{\widehat{\sigma}_{max}\,(1-\varepsilon+\frac{\varepsilon}{\rho_e})} \qquad (1)$$

where B_b and N_e^b are the set and the number of elements contained in block b, ρ_e is the relative density of the element e and ε is the relaxation parameter. According to that, the number of constraints of the optimization problem is equal to the number of blocks. The normalized stress $\widehat{\sigma}_e^{*}$ is obtained by dividing the reference stress $\widehat{\sigma}$ between the maximum stress allowable multiplied by the relaxation factor. The aggregation parameter μ must be higher than 20 [2].

This formulation produces very well defined material configurations and requires only a bit more computing resources than the use of only one global constraint. However, it is necessary to decide an appro-

priate way of aggregating the elements in blocks, because this distribution modifies the corresponding stress constraints. However, we have observed in the examples solved that the block configuration is not crucial to obtain satisfactory solutions. It is much more relevant to define an adequate number of blocks or to use a high enough value of μ. In this paper, we have defined the block configuration following the numbers of the elements in the mesh of Finite Elements.

As application example we show the solution obtained with this formulation for the Michell cantilever beam. The dimensions of the beam can be observed in figure 1. The beam is supported along the contour of the left hole and supports a vertical force of 6 kN. The material is steel with elastic limit $\widehat{\sigma}_{max} = 230$ MPa, Young's Modulus $E = 2.1\ 10^5$ MPa and Poisson's ratio $\nu = 0.3$. The mass density is $\gamma_{mat} = 7650$ kg/m^3 and the thickness is 0.01 m. The mesh is defined by 6400 8-node elements aggregated in 80 blocks.

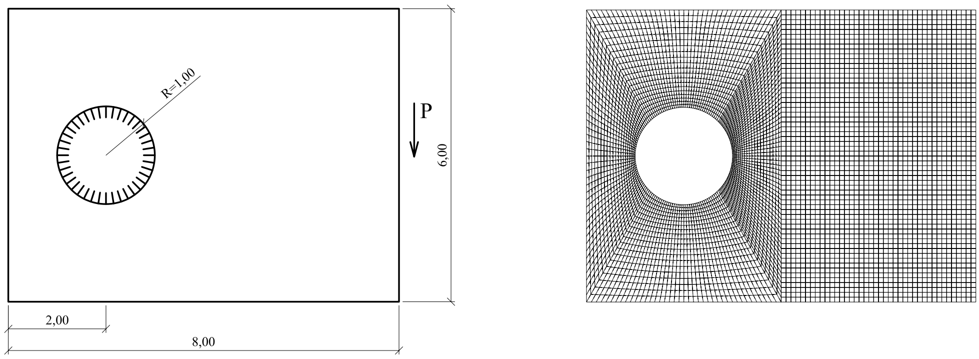

Figure 1: Michell cantilever beam (units in meters).

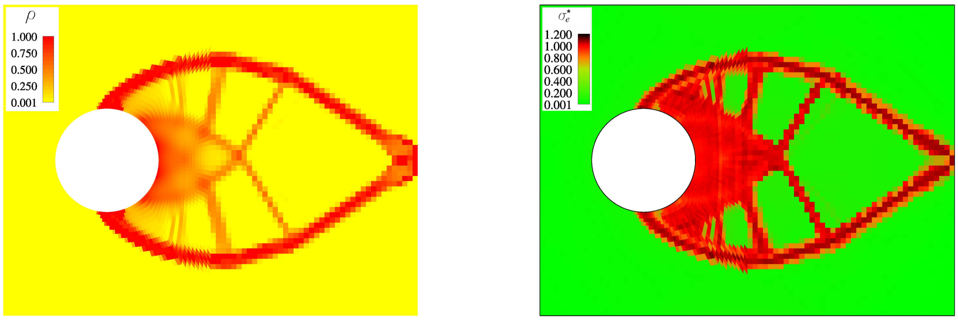

Figure 2: Optimum design (left) and normalized stress configuration (right). ($\varepsilon = 0.01$, $\mu = 40$, $p = 4$)

REFERENCES

[1] P. Duysinx and M.P. Bendsøe. "Topology optimization of continuum structures with local stress constraints", *Int. J. for Numerical Methods in Engineering*, **43**, 1453–1478, 1998.

[2] J. París, M. Casteleiro, F. Navarrina & I. Colominas. "Topology optimization of structures with local and global stress constraints", *Int. Conference on Computational & Experimental Engineering and Sciences*, ICCES: Miami, 2007.

[3] F. Navarrina, I. Muíños, I. Colominas and M. Casteleiro. "Topology optimization of structures: a minimum weight approach with stress constraints", *Advances in Engineering Software*, **36**, 599–606, 2005.

[4] M.P. Bendsøe. *Optimization of structural topology, shape, and material*, Springer, 1995.