8th. World Congress on Computational Mechanics (WCCM8)
5th European Congress on Computational Methods in Applied Sciences and Engineering (ECCOMAS 2008)
June 30 –July 5, 2008
Venice, Italy

AN FMM FOR PERIODIC SCATTERING PROBLEMS IN ELECTROMAGNETICS

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Key Words: *Boundary Element Method, Fast Multipole Method, Periodic problems, Maxwell equations*

ABSTRACT

New optical structures in which periodicity plays a significant role are being developed these days. One of most remarkable examples of such structures is the photonic crystal. Photonic crystals are composed of periodic dielectric or metallic structures. By designing the periodic structure properly, we can make band gaps in photonic crystals: we can prohibit propagation of light within certain ranges of frequencies called band gaps. In addition, defects in the periodicity can cause localised modes in the vicinity of defects, which may lead to a pass band in a band gap. Photonic crystals thus enable us to control light freely since we can guide or store light using these phenomena. Nowadays many researchers make great efforts to fabricate new optical devices using photonic crystals: such devices include zero-threshold lasers, large scale optical integrated circuits etc.

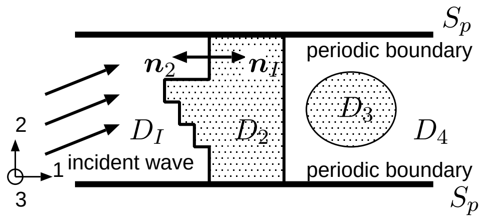

Figure 1: Periodic boundary value problems

Considering such applications, it is concluded very important to develop designing tools for periodic structures, especially in dynamics. Indeed, large scale analyses are required in the design of optical devices such as photonic crystals, since shapes of actual optic devices are very complicated. FM-BIEMs (Fast Multipole Boundary Integral Equation Methods)[1,2] are good candidates as fast solvers of large scale wave problems since FM-BIEMs require only $O(N(\log N)^{\alpha})$ operations in problems with N boundary elements. However, we can find few researches on large scale periodic scattering problems other than the works of the present author's group[3]. In view of these backgrounds, we develop an FMM for periodic problems for scattering problems in electromagnetics in the present study. The target problems are doubly periodic problems in Maxwell's equations in 3D in frequency domain.

Henceforth we express the formulation of periodic scattering problems for Maxwell's equations in 3D. Let D be the domain defined by $D = (-\infty, \infty) \otimes (-L/2, L/2) \otimes (-L/2, L/2)$ which is further subdivided into N subdomains $D = \overline{D_1 \cup D_2 \cup \cdots \cup D_N}$ (Figure 1). In each of the subdomains D_i we assume that the following Maxwell's equations are satisfied:

$$\nabla \times \boldsymbol{E} = i\omega\mu^i \boldsymbol{H}, \quad \nabla \times \boldsymbol{H} = -i\omega\epsilon^i \boldsymbol{E} \quad \text{in } D_i,$$

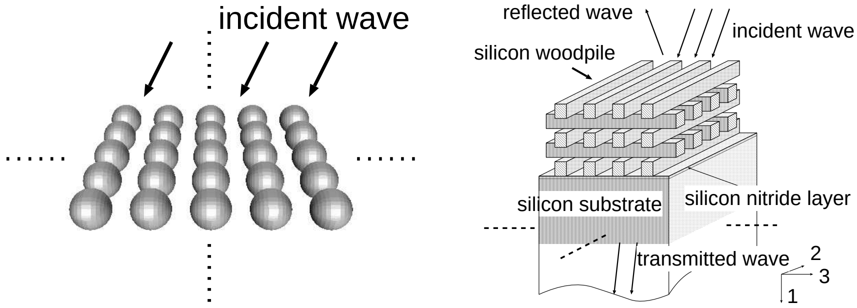

Figure 3: Examples of models. Left: 2 dimensional array of dielectric spheres, Right: Woodpile crystal

where ω is the frequency (with the $e^{-i\omega t}$ time dependence), ϵ^i and μ^i are the dielectric constant and the magnetic permeability for the material occupying D_i. In the subdomain which extends to $x_1 \to -\infty$, we consider the incident plane wave. On interfaces between different subdomains we impose the continuity conditions on the tangential components of $\boldsymbol{E}$ and $\boldsymbol{H}$. On the periodic boundaries given by $S_p = \{\boldsymbol{x} | \ \boldsymbol{x} \in \partial D, \ |x_2| = L/2 \text{ or } |x_3| = L/2\}$ we require the following periodic boundary conditions:

$$\boldsymbol{E}(x_1, L/2, x_3) = e^{i\beta_2}\boldsymbol{E}(x_1, -L/2, x_3), \quad \boldsymbol{E}(x_1, x_2, L/2) = e^{i\beta_3}\boldsymbol{E}(x_1, x_2, -L/2),$$
$$\boldsymbol{H}(x_1, L/2, x_3) = e^{i\beta_2}\boldsymbol{H}(x_1, -L/2, x_3), \quad \boldsymbol{H}(x_1, x_2, L/2) = e^{i\beta_3}\boldsymbol{H}(x_1, x_2, -L/2),$$

where β_i is the phase difference of the incident wave at $x_i = -L/2$ and $x_i = L/2$, expressed by $\beta_i = Lk_i^{\text{inc}}$ $(i = 2, 3)$, and k_i^{inc} is the wave number vector of the incident wave.

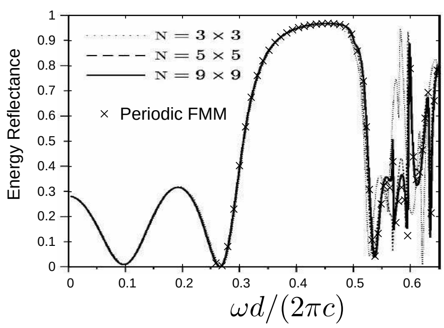

Figure 2: Reflectance of the woodpile crystal (d: distance between the centres of woodpiles in the x_2 or x_3 direction, ω: frequency, c: light velocity)

In this study we deal with standard and important models in the field of photonic crystals. We show examples of the models considered in the present study in Figure 3, where the left figure shows a model of slab photonic crystals and the right figure gives a model of woodpile photonic crystals. In the case of the woodpile crystal, we computed the energy reflectance and compared the results obtained with the present method with those reported by Gralak et al[4]. We plot the energy reflectance for various wave numbers in Figure 2. As seen in Figure 2, our results agreed well with the most accurate results, denoted by '$N = 9 \times 9$' (solid line) obtained by Gralak et al.

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