8th. World Congress on Computational Mechanics (WCCM8)
5th European Congress on Computational Methods in Applied Sciences and Engineeering (ECCOMAS 2008)
June 30 –July 5, 2008
Venice, Italy

Meshes using a 3d anisotropic metric

*** C.Dobrzynski[1], V. Ducrot[2] and P. Frey[2,3]**

[1]Univ. Bordeaux I, UMR 5251
INRIA, Scalapplix team
Talence, France

[2]UPMC Paris 06, UMR 7528
Laboratoire J.L. Lions
Paris, France

[3]Universidad de Chile, UMI 2806
Centro de Modelamiento Matemático
Santiago, Chile

Key Words: *anisotropy, mesh adaptation, moving mesh, levelset, elasticity problem.*

ABSTRACT

In many unsteady simulations the geometry of a computational domain undergoes large deformation or displacement. For instance, in optimization, the object shape has an influence on the flow characteristics and so the mesh must be adapted to this domain geometry. Another domain of application concerns the fluid-structure interactions where the mesh has to account for moving bodies.

We are interested in techniques for coupling moving meshes and anisotropic mesh adaptation in order to increase, or at least to preserve, the solution accuracy in unsteady simulations. Here, we propose two different types of applications: the capture of interfaces with anisotropic meshes and an *r-method* based on a linear elasticity equation for prescribed the displacement field.

In the two approaches, the mesh anisotropy is defined by a metric tensor field supplied at the mesh vertices. This metric field is designed to prescribe the size and the streching of the element based on an edge length control. We propose to use local mesh modifications for the generation of anisotropic tetrahedral meshes. The specificity of our remeshing process is related to the manner a new point is inserted on the current mesh, using an anisotropic extension of the classical Delaunay technique [1].

Interface tracking using an anisotropic metric. In this application, we introduce a geometric control of the piecewise geometric approximation of interfaces using an anisotropic metric [2]. It relies on the generation and the adaptation of a triangulation to an anisotropic metric tensor field related to the intrinsic properties of the manifold, namely its local curvatures. This method can be used to deal with evolving interfaces like those encountered in multi-domains or multi-fluids simulations using levelsets.

For instance, we consider here a problem where the interface (the boundary between two domains) is defined using a levelset formulation. Typically, the problem we face is to construct an accurate piecewise linear approximation of the interface denoted Γ. To the end, the aim is to generate a mesh for which the distance between any vertex and the surface is bounded. Let ε denote the desired accuracy, we define the following metric (in three dimensions) at the mesh vertices for any triangle intersected the manifold Γ:

$$\mathcal{M} = \mathcal{R} \begin{pmatrix} 1/\varepsilon^2 & 0 & 0 \\ 0 & |\lambda_2|/\varepsilon & 0 \\ 0 & 0 & |\lambda_3|/\varepsilon \end{pmatrix} {}^t\mathcal{R},$$

where $\mathcal{R}$ represents a rotation matrix. If the interface Γ corresponds to an isovalue (for instance $u = 0$) of a levelset function u defines on a domain $\Omega \subset \mathbb{R}^3$ then the λ_i represents the eigenvalues of Du, the

Hessian of u. Moreover if (v_1, v_2) is a basis of the tangent plane to the surface at the vertex then the rotation matrix will be $R = (Du\, v_1\, v_2)$. At any the other mesh vertex (not in a triangle intersected by Γ), we prescribe a metric αI_3 where $\alpha > 0$ is a scalar value used to provided a nice mesh gradation (α is proportionnal to the distance to the interface). Using this metric tensor, highly anisotropic elements can be created in the vicinity of the interface.

To illustrate this approach, we generated an anisotropic mesh near an interface corresponding to the following surface Σ defined on $\Omega = [-1; 1]^3$ by $r = \cos(6\theta)\, \cos(3\phi)\, \cos(6\phi)$, where $\theta \in [0; 2\pi]$ and $\phi \in [-\frac{\pi}{2}; \frac{\pi}{2}]$. Figure 1 shows the surface and the volumic tetrahedra cut.

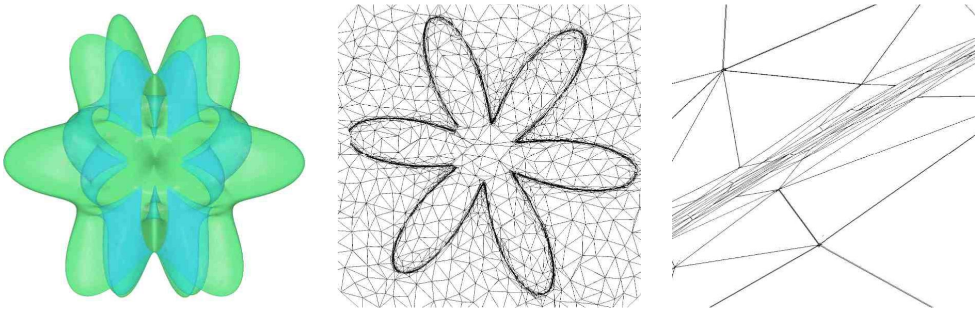

Figure 1: *Isosurfaces, volumic cut and zoom on the anisotropic region for the analytical surface* Σ.

Rigid bodies displacement. We address the displacement of rigid bodies moving through a computational domain discretize using a tetrahedral mesh using an *r-method*. The displacement field is given as a solution of a linear elasticity problem [3] and the mesh is adapted at each time step using local mesh modifications. In addition, we propose to couple rigid bodies displacements and anisotropic mesh adaptation methods.

Given a displacement on a part of the domain boundary, our aim is to generate a new mesh in which the domain boundary has moved. Let Ω be a computational domain and let $\Gamma = \Gamma_f \cup \Gamma_m$ be its boundary ; Γ_f represents the fixed part of the boundary and Γ_m is the moving part. A displacement v_o is prescribed on Γ_m and the solution of the linear elasticity equation will provided the displacement on each mesh vertex inside the domain. Solving this type of equation allows us to keep the number of nodes unchanged throughout the all simulation. This procedure is fully automatic [1]. Figure 2 shows an example.

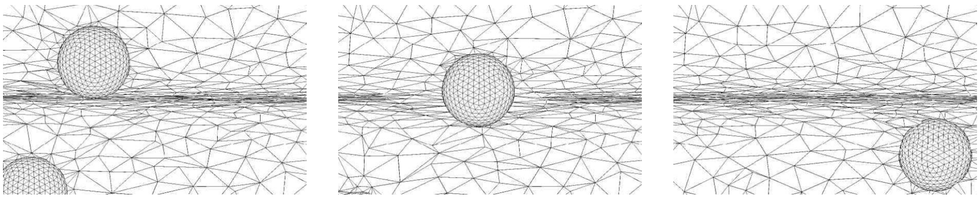

Figure 2: *Example of rigid bodies displacement (volumic cut).*

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