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A SECOND ORDER CELL-CENTERED LAGRANGIAN SCHEME FOR TWO-DIMENSIONAL COMPRESSIBLE FLOW PROBLEMS

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ABSTRACT

The physical model that is considered throughout this paper is based on the equations of fluid dynamics written in Lagrangian form. This form is well adapted to the simulation of multi-material compressible fluid flows, such as those encountered in the domain of Inertial Confinement Fusion (ICF), see [1]. Our aim is to develop a new high order Lagrangian cell-centered scheme for two-dimensional gas dynamics equations on unstructured meshes.

Here, we propose a new Lagrangian cell-centered scheme in which the vertex velocities and the numerical fluxes through the cell interfaces are not computed independently as usual but in a consistent manner with an original solver located at the nodes, see [3]. The primary variables of our scheme are the specific volume, the velocity and the total energy. The main new feature of the algorithm is the introduction of four pressures on each edge, two for each node on each side of the edge. This extra degree of freedom allows us to construct a nodal solver that fulfills two properties. First, the conservation of momentum and total energy is ensured. Second, a semi-discrete entropy inequality is provided. We show, in the limit of a one-dimensional flow computed by our two-dimensional solver, or for flows in a cylindrical geometry, that the scheme recovers the classical Godunov approximate Riemann solver. It is interesting to realize that we only need the knowledge of the isentropic speed of sound: it is very easy to extend it to more general equation of state. The precise form of the equation of state, analytical or tabulated, does not matter. The boundary conditions are taken into account in a natural way. This unstructured scheme is only first order in time and space, however it appears to be quite robust and versatile according to the numerical results obtained for the various test cases presented in [3].

We have developed a second order extension of our scheme, based on a MUSCL type approach. In order to compute the fluxes more accurately we use a piecewise linear representation of the pressure and the velocity field. This linear two-dimensional reconstruction is obtained via a least squares procedure, see [2]. The monotonicity of the reconstructed fields is ensured using post-reconstruction monotone limiters. For the pressure we use a classical scalar limiter that reduces to the classical minmod limiter for one-dimensional flows. For the velocity field we design a new matricial limiter for the tensor gradient, namely the components of the velocity are limited simultaneously. The second order accuracy for the time discretization is achieved using a classical Runge-Kutta two steps procedure. We show on various numerical test cases the robustness and the accuracy of this second order scheme, see Figure 1.

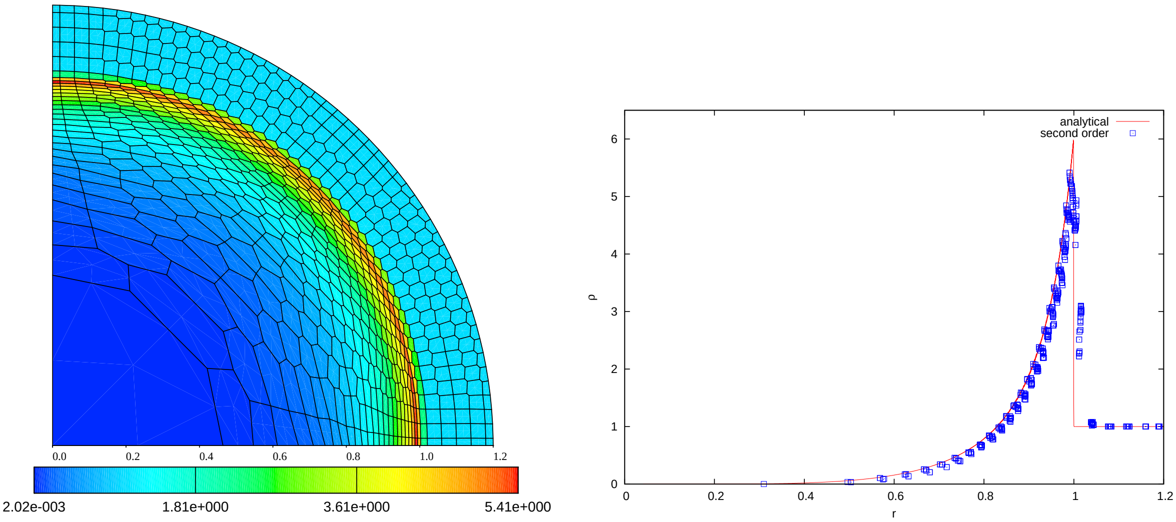

Figure 1: Sedov blastwave on a polygonal grid: density map (left) and density in all the cells (right) at $t = 1$.

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