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Comparison of mixed and nodal based formulations for linear tetrahedral elements in elasticity and plasticity

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ABSTRACT

This work concentrates on the comparison of two competing strategies for tetrahedral elements, mixed enhanced elements and nodal based elements. As a starting point for the former (mixed finite elements) a bilinear formulation - linear for displacements and linear for pressure - is used in the finite element discretization. In order to overcome the oscillatory effect of the bilinear concept, two well known concepts are introduced: Firstly, the method of incompatible modes, originally by WILSON *et al.* for quadrilateral elements and secondly, the enhanced assumed strain method (EAS) given in SIMO and RIFAI. Both formulations have been combined to volume bubble [3] and area bubble functions in [1], in order to improve the poor performance of linear tetrahedral elements. Furthermore, in [2] the concepts of incompatible modes and of enhanced assumed strains have been employed for stabilization, which becomes most attractive for implementation of nonlinear problems.

Furthermore, DOHRMANN *et al.* in ref. [4] described the basic idea of the nodal based formulation for linear tetrahedrons in linear elasticity. PUSO and SOLBERG [5] improve this formulation by adding a stabilization scheme relevant for eigenvalue and large deformations in elastoplasticity. In order to overcome volume locking and to reduce simultaneously shear locking, new larger volumes around each node of the initial tetrahedral mesh are introduced. The strain is assumed to be constant. Thus, no more incompressibility constraints than nodes in 3-D tetrahedral meshes are introduced, whereas classically such meshes present more elements than nodes, leading thus to more incompressibility constraints as needed. In this work we extend the works of DOHRMANN [4] *et al.* and PUSO and SOLBERG [5] to thermo-elastoplasticity and by adopting Gauss points to integrate more accurately the temperature dependent constitutive laws. This extension is needed in presence of strong variations of the thermophysical properties, like at the solidification front in the thermomechanical simulation of casting processes.

Additionally, we compare mixed and nodal based formulations for linear tetrahedral elements in elasticity and plasticity in the numerical examples. Cook's membrane problem is investigated as a first benchmark for the cases of compressible (Poisson coefficient $\nu = 0.33$) and near incompressible elasticity ($\nu = 0.499995$). Next, this test is also performed in elastoplasticity with isotropic hardening. The geometry and the finite element discretization are described in [1].

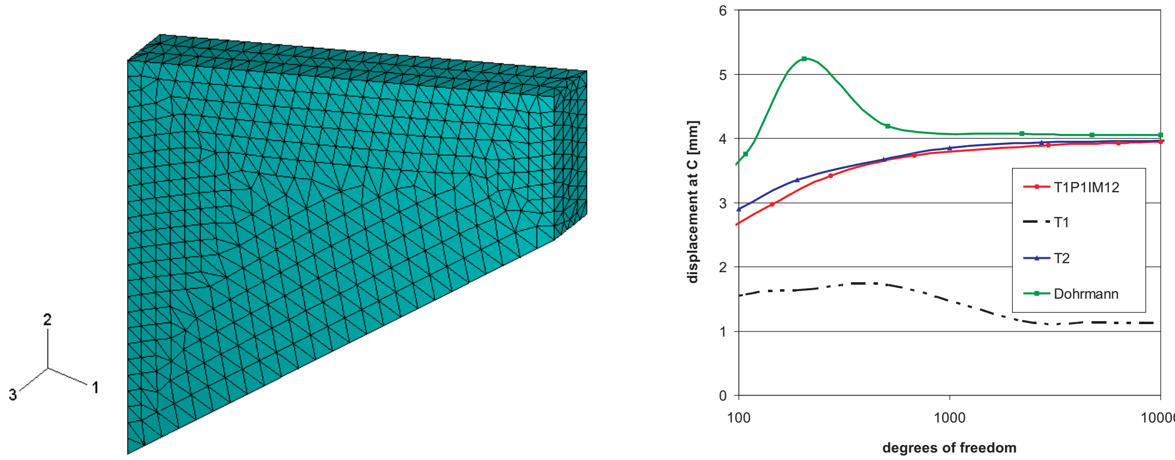

Figure 1: Cook's membrane problem: Left: Discretization with tetrahedral elements. Right: Vertical tip displacement for linear ($T1$), quadratic ($T2$), area bubble ($T1P1IM12$) and nodal based elements (Dohrmann) with $\nu \approx 0.5$.

In Figure 1 the fine discretization and the convergence of the vertical tip displacement are illustrated for the test case $\nu \approx 0.5$ in elasticity. In the diagram on the right side convergence of the linear (T1) and quadratic (T2) elements, the new mixed element with area bubbles $T2P1IM12$ and the nodal based Dohrmann elements are depicted. Both concepts, the mixed enhanced and the nodal based elements converge to the correct tip displacement. For coarser meshes the nodal based element converges like a stress element, whereas the linear element $T1$ is too stiff and does not converge. The stress distribution along the clamped edge was also analyzed, see e.g. [1,2]. The quadratic elements $T2$ gives an oscillatory behavior which is also present, at a reduced level, for the Dohrmann [4] element without stabilization. This result indicates clearly the necessity to implement PUSO and SOLBERG's [5] stabilization term. The damping effect of the stress oscillations for the area bubble elements is however excellent, thus illustrating the good stabilization of the incompatible modes and of the enhanced strains. Finally in thermo-elastoplasticity, a cylinder under internal pressure and temperature gradient is investigated. The obtained stress distributions with the different tetrahedron improvements are compared with reference solutions of the literature obtained with quadrangular axisymmetric meshes.

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