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PREDICTIVE FRAMEWORK FOR MULTISCALE COMPUTATIONS IN GRANULAR MEDIA

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ABSTRACT

In this work, a novel predictive framework for the multiscale modeling of granular materials is presented. The multiscale procedure is hierarchical and hinges crucially on a suitable information-passing scheme. The idea is to *probe* the grain scale—using unit cell computations—to extract key material parameters such as dilation and friction angles.

For simplicity, the starting point is a perfectly-plastic generalized Druker-Prager model [1]. In this case, the elastoplastic constitutive tensor is given by

$$\boldsymbol{c}^{\mathrm{ep}} = \boldsymbol{c}^{\mathrm{e}} - \frac{\boldsymbol{c}^{\mathrm{e}} : \boldsymbol{g} \otimes \boldsymbol{f} : \boldsymbol{c}^{\mathrm{e}}}{\boldsymbol{g} : \boldsymbol{c}^{\mathrm{e}} : \boldsymbol{f}} \tag{1}$$

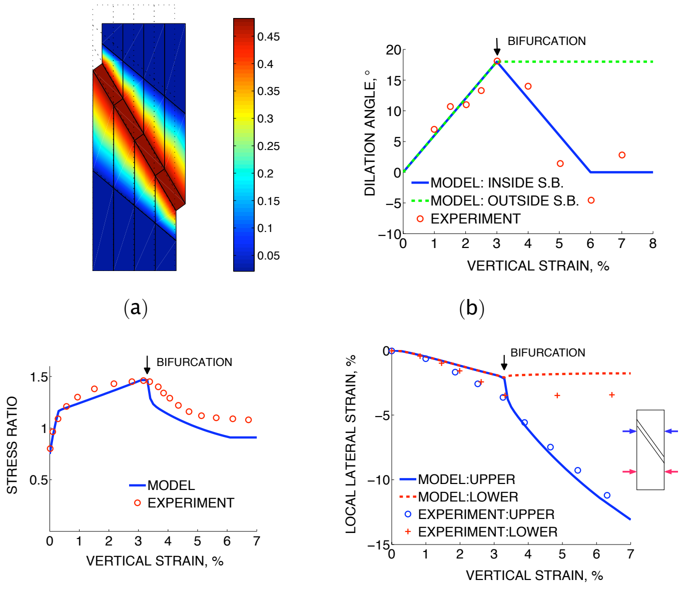
where $\boldsymbol{c}^{\mathrm{e}}$ is the (linear) elastic constitutive tensor, and

$$\begin{aligned} \boldsymbol{g} := \partial G / \partial \boldsymbol{\sigma} &= 1/3 \beta \mathbf{1} + \sqrt{3/2}\, \hat{\boldsymbol{n}} && (2) \\ \boldsymbol{f} := \partial F / \partial \boldsymbol{\sigma} &= 1/3 \mu \mathbf{1} + \sqrt{3/2}\, \hat{\boldsymbol{n}} && (3) \end{aligned}$$

The tensor $\mathbf{1}$ is the identity and $\hat{\boldsymbol{n}} = \boldsymbol{s}/\|\boldsymbol{s}\|$, where $\boldsymbol{s}$ is the deviatoric component of the stress tensor $\boldsymbol{\sigma}$. The tensors $\boldsymbol{f}$ and $\boldsymbol{g}$ represent the gradients of the yield surface F and plastic potential G in stress space, respectively. Phenomenology is typically introduced when determining the current values (and evolution) of the friction coefficient μ and the dilatancy β.

The main contribution of this work is the formulation of unit cell computations to extract the plasticity parameters μ and β using direct numerical simulations. In this fashion, the constitutive framework is explicitly constrained by the grain scale mechanisms.

Equally important is the demonstration of the predictive capabilities of the method to capture macroscopic inhomogeneous behavior in solids. To this end, the framework is used to predict the behavior of a dense sand undergoing strain localization under plane strain compression. Physical experiments conducted at Northwestern University used 2D-DIC techniques to extract local changes in dilation angles observed in shear bands in sand specimens [2]. Figure 1 shows an example of this technique. As shown in the figure, a sample of dense sand was failed under plane strain compression. High-resolution digital pictures were taken during the experiment that allowed 2D-DIC to obtain displacement fields, which were used to calculate dilation angles. In the multiscale framework, only the dilation angle (Figure 1(b)) is taken as an input material parameter (using the experiments themselves as unit cell computations). Young's modulus and Poisson's ratio are constant throughout the simulation. Furthermore, the friction angle is updated using $\mu = \beta + \mu_{cs}$, where μ_{cs} is the friction angle at isochoric plastic flow—a material constant. Figure 1 shows the results of the simulation compared with the experimental results. The accuracy of the method is apparent.



(a) (b)

(c) (d)

Figure 1: Validation of multiscale framework: (a) finite element solution of plane strain compression test in dense sand, (b) dilation angles as a function of deformation measured using 2D-DIC, (c) load-displacement histories, and (d) local lateral strains showing onset of inhomogeneous deformations. After [1].

Future research will combine this technique with XFEM to propagate shear bands without aligning the finite element mesh [3]. This predictive multiscale model opens the door to more realistic simulation of granular media, replacing phenomenology with grain scale direct numerical simulations.

REFERENCES

[1] J. E. Andrade, X. Tu, and R. J. Finno. "Multiscale framework for behavior prediction in granular media". In preparation, 2008.

[2] M. A. Mooney, R. J. Finno, and M. G. Viggiani. "A unique critical state for sand?". *Journal of Geotechnical and Geoenvironmental Engineering*, 124:1100–1108, 1998.

[3] T. Belytshcko, S. Loehnert, and J. H. Song. "Multiscale aggregating discontinuities: a method for circumventing loss of material stability". *International Journal for Numerical Methods in Engineering*, In press, 2007.