

Numerical Treatment of Shallow Water Equations with Uncertain Parameters

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ABSTRACT

The shallow water equations (SWE) are often taken as a mathematical model for the simulation of flood flows, but some parameters in the SWE which can substantially affect the simulation are associated with uncertainties, e.g. upstream hydrograph, flood plain topography, and friction coefficient. Incorporating these uncertainties into the SWE gives rise to *stochastic* shallow water equations (SSWE), which provides a new possibility of more robust and informative flood simulation, while the resulting high number of stochastic dimensions also poses a computational challenge.

This work addresses uncertainty quantification in numerical flood modelling with SSWE on a real test case of a 1982 flood of the Júcar river in Spain as it broke the Tous dam (La Pantanada de Tous). Numerical approaches including the Monte Carlo method, the stochastic collocation method with full and sparse tensor products, and a novel *virtual sampling* Monte Carlo method are applied to the test case of uncertainty quantification in flood modelling with 8–20 independent random variables.

The hydraulic and topographical data of the 1982 Tous flooding is provided in [1], which also describes the effects of the flood and allows detailed comparisons of numerical models with real data. A finite volume solver for shallow water equations [2] is used in this work with the developer's authorisation.

We compare the plain Monte Carlo—also as a base case—with stochastic collocation and a newly proposed virtual sampling Monte Carlo method. This latter method makes a novel exploitation of the probability density function of the independent random variables, and seems to be superior to the plain Monte Carlo method. In the comparison of these stochastic computational methods, particular emphasis is placed on the behaviour depending on what kind of result or results have to be computed, what accuracy is desired, and on the asymptotic behaviour with increasing numbers of random variables or stochastic dimensions.

References

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- [2] S. Soares Frazao. *Dam-break induced flow in complex topographies, theoretical, numerical and experimental approaches*. PhD thesis, University Catholique de Louvain, Louvain-la-Neuve, Belgium, 2002.